LESSON 11.2

Solving Linear Systems by Substitution

Common Core Math Standards

The student is expected to:

COMMON CORE **A-REI.C.6**

Solve systems of linear equations exactly... focusing on pairs of linear equations in two variables.

Mathematical Practices

COMMON CORE **MP.6 Precision**

Language Objective

Explain to a partner how to solve a system of linear equations by substitution.

ENGAGE

Essential Question: How can you solve a system of linear equations by using substitution?

Solve one equation for one variable and substitute the resulting expression into the other equation. Solve for the value of the other variable in that equation, and then substitute that value into either equation to find the value of the first variable.

PREVIEW: LESSON PERFORMANCE TASK

View the Engage section online. Discuss why a manufacturer might choose to produce either a cheaper version or a more expensive version of a product. Then preview the Lesson Performance Task.

Name ______________ Class ______________ Date ______________

11.2 Solving Linear Systems by Substitution

Essential Question: How can you solve a system of linear equations by using substitution?

Resource Locker

Explore Exploring the Substitution Method of Solving Linear Systems

Another method to solve a linear system is by using the substitution method.

In the system of linear equations shown, the value of y is given. Use this value of y to find the value of x and the solution of the system.

$$\begin{cases} y = 2 \\ x + y = 6 \end{cases}$$

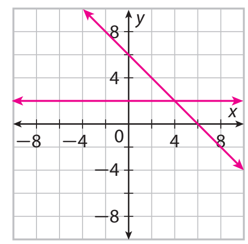
(A) Substitute the value of y in the second equation and solve for x.

$$x + y = 6$$
$$x + \boxed{2} = 6$$
$$x = \boxed{4}$$

(B) The values of x and y are known. What is the solution of the system?

Solution: $(\boxed{4}, \boxed{2})$

(C) Graph the system of linear equations. How do your solutions compare?
The solutions are the same.



(D) Use substitution to find the values of x and y in this system of linear equations. Substitute $4x$ for y in the second equation and solve for x. Once you find the value for x, substitute it into either original equation to find the value for y.

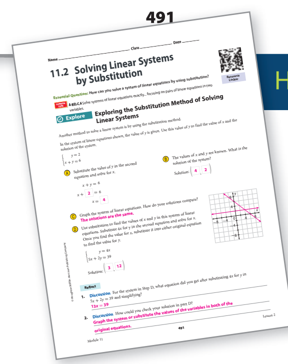
$$\begin{cases} y = 4x \\ 5x + 2y = 39 \end{cases}$$

Solution: $(\boxed{3}, \boxed{12})$

Reflect

1. **Discussion** For the system in Step D, what equation did you get after substituting $4x$ for y in $5x + 2y = 39$ and simplifying?
$13x = 39$

2. **Discussion** How could you check your solution in part D?
Graph the system or substitute the values of the variables in both of the original equations.



HARDCOVER

Turn to Lesson 11.2 in the hardcover edition.

Explain 1 Solving Consistent, Independent Linear Systems by Substitution

The **substitution method** is used to solve a system of equations by solving an equation for one variable and substituting the resulting expression into the other equation. The steps for the substitution method are as shown.

1. Solve one of the equations for one of its variables.
2. Substitute the expression from Step 1 into the other equation and solve for the other variable.
3. Substitute the value from Step 2 into either original equation and solve to find the value of the other variable.

Example 1 **Solve each system of linear equations by substitution.**

(A) $\begin{cases} 3x + y = -3 \\ -2x + y = 7 \end{cases}$

Solve an equation for one variable.

$3x + y = -3$	Select one of the equations.
$y = -3x - 3$	Solve for y. Isolate y on one side.

Substitute the expression for y in the other equation and solve.

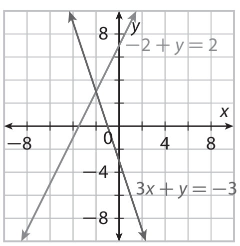
$-2x + (-3x - 3) = 7$	Substitute the expression for y.
$-5x - 3 = 7$	Combine like terms.
$-5x = 10$	Add 3 to both sides.
$x = -2$	Divide each side by -5.

Substitute the value for x into one of the equations and solve for y.

$3(-2) + y = -3$	Substitute the value of x into the first equation.
$-6 + y = -3$	Simplify.
$y = 3$	Add 6 to both sides.

So, $(-2, 3)$ is the solution of the system.

Check the solution by graphing.



$3x + y = -3$	$-2x + y = 7$
x-intercept: -1	x-intercept: $-\frac{7}{2}$
y-intercept: -3	y-intercept: 7

The point of intersection is $(-2, 3)$.

EXPLORE

Exploring the Substitution Method of Solving Linear Systems

INTEGRATE TECHNOLOGY

Have students use the graphing tools available in graphing calculators or online to check the solution to a system of linear equations.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Modeling

MP.4 Make sure that students understand the connection between a system of linear equations and its graph. The intersection of the two lines shows the solution of the system of equations.

EXPLAIN 1

Solving Consistent, Independent Linear Systems by Substitution

QUESTIONING STRATEGIES

How do you choose which equation you solve first and which variable you solve it for? Explain. **Look for an equation that can easily be solved for one variable, such as an equation in which one variable has a coefficient of 1 or −1. The solution will be the same no matter which equation you solve first, but this will make the process easier.**

PROFESSIONAL DEVELOPMENT

Learning Progressions

In this lesson, students continue their work with systems of linear equations. Having learned how to solve a system by graphing, they now learn how to solve a system algebraically by using the substitution method. They learn how to determine whether a system has zero, one, or infinitely many solutions, as well as how to use systems of linear equations to model real-world situations. As they continue, students will learn other algebraic methods for solving systems of linear equations, and will learn how to decide which approach is more efficient for a given system.

AVOID COMMON ERRORS

Make sure students understand that after you find the value of one variable, you must also solve for the other variable. Some students may consider their work done when they have evaluated one variable.

Ⓑ $\begin{cases} x - 3y = 9 \\ x + 4y = 2 \end{cases}$

Solve an equation for one variable.

$x - 3y = 9$	Select one of the equations.
$x = \boxed{3y + 9}$	Solve for x. Isolate x on one side.

Substitute the expression for $\underline{x}$ in the other equation and solve.

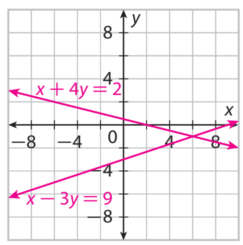
$\left(\boxed{3y + 9}\right) + 4y = 2$	Substitute the expression for $\underline{x}$.
$\boxed{7y + 9} = 2$	Combine like terms.
$\boxed{7y} = \boxed{-7}$	Subtract $\boxed{9}$ from both sides.
$y = \boxed{-1}$	Divide each side by $\boxed{7}$.

Substitute the value for y into one of the equations and solve for x.

$x - 3\left(\boxed{-1}\right) = 9$	Substitute the value of y into the first equation.
$\boxed{x + 3} = 9$	Simplify.
$x = \boxed{6}$	Subtract $\boxed{3}$ from both sides.

So, $\left(\boxed{6}, \boxed{-1}\right)$ is the solution by graphing.

Check the solution by graphing.



$x - 3y = 9$	$x + 4y = 2$
x-intercept: $\boxed{9}$	x-intercept: $\boxed{2}$
y-intercept: $\boxed{-3}$	y-intercept: $\boxed{\frac{1}{2}}$

The point of intersection is $\left(\boxed{6}, \boxed{-1}\right)$.

Reflect

3. Explain how a system in which one of the equations if of the form $y = c$, where c is a constant is a special case of the substitution method.

There is no need to solve for y in terms of x because the value of y is already known.

4. Is it more efficient to solve $-2x + y = 7$ for x than for y? Explain.

No, because more steps are needed and $x = \frac{y}{2} - \frac{7}{2}$, which is more difficult to substitute than $y = 2x + 7$.

COLLABORATIVE LEARNING

Peer-to-Peer Activity

Group students in pairs, and give each pair a system of linear equations. Have one student solve the first equation for y and the other solve the second equation for y. Then have both students continue to solve independently using substitution. Each should arrive at the same solution. Have partners compare their work and discuss which substitution is more efficient.

Your Turn

5. Solve the system of linear equations by substitution.

$\begin{cases} 3x + y = 14 \\ 2x - 6y = -24 \end{cases}$ $y = -3x + 14$ $2x - 6(-3x + 14) = -24$ $x = 3$ $3(3) + y = 14$ $y = 5$ The solution of the system is $(3, 5)$.

Explain 2 Solving Special Linear Systems by Substitution

You can use the substitution method for systems of linear equations that have infinitely many solutions and for systems that have no solutions.

Example 2 **Solve each system of linear equations by substitution.**

Ⓐ $\begin{cases} x + y = 4 \\ -x - y = 6 \end{cases}$

Solve $x + y = 4$ for x.

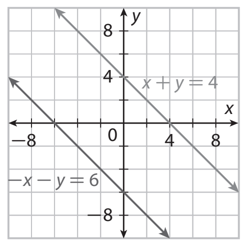
$x = -y + 4$

Substitute the resulting expression into the other equation and solve.

$-(-y + 4) - y = 6$ Substitute.

$-4 = 6$ Simplify.

The resulting equation is false, so the system has no solutions.



The graph shows that the lines are parallel and do not intersect.

Ⓑ $\begin{cases} x - 3y = 6 \\ 4x - 12y = 24 \end{cases}$

Solve $x - 3y = 6$ for x.

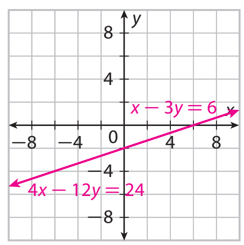
$x = \boxed{3y + 6}$

Substitute the resulting expression into the other equation and solve.

$4\left(\boxed{3y + 6}\right) - 12y = 24$ Substitute.

$\boxed{24} = 24$ Simplify.

The resulting equation is true, so the system has infinitely many solutions.



The graphs are the same line, so the system has infinitely many solutions.

Reflect

6. Provide two possible solutions of the system in Example 2B. How are all the solutions of this system related to one another?

Sample solutions: $(0, -2)$ and $(6, 0)$; all the solutions of this system are points on the line.

EXPLAIN 2

Solving Special Linear Systems by Substitution

QUESTIONING STRATEGIES

When solving a system of linear equations by substitution, how can you tell if the system has no solution or infinitely many solutions? If it has no solutions, the solution process will result in an equation that is false. If it has infinitely many solutions, the solution process will result in an equation that is always true.

How does the graph of a system of linear equations tell you it has no solution or infinitely many solutions? When the equations represent two parallel lines, the lines do not intersect, so there is no solution. When both equations represent the same line, the system has infinitely many solutions.

AVOID COMMON ERRORS

Make sure students understand that when you substitute an expression for a variable, the expression should be placed inside parentheses. Remind students to follow the order of operations and to apply the Distributive Property correctly when dealing with expressions inside parentheses.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Critical Thinking

MP.3 You can use algebra tiles to model and solve some systems of linear equations. Solve for one variable using the first equation, then model the second equation.

EXPLAIN 3

Solving Linear System Models by Substitution

AVOID COMMON ERRORS

Some students may struggle with solving by substitution because they automatically start by solving the first equation for y. Encourage them to look at both equations and check whether any of the variables has a coefficient of 1 or -1. Then have students solve for that variable first.

QUESTIONING STRATEGIES

Is it more accurate to check your solution by graphing or by substituting back into the original equations? Explain. **Substituting, because if the solution does not consist of integers, graphing may not give an accurate check.**

INTEGRATE MATHEMATICAL PRACTICES

Focus on Technology

MP.5 Some real-world problems, especially those involving money, may have systems of equations with decimal coefficients. It is far easier to use a graphing calculator to check the solution than it is to draw the graph by hand.

Your Turn

Solve each system of linear equations by substitution.

7. $\begin{cases} -2x + 14y = -28 \\ x - 7y = 14 \end{cases}$

$x = 7y + 14$

$-2(7y + 14) + 14y = -28$

$-28 = -28$

infinitely many solutions

8. $\begin{cases} -3x + y = 12 \\ 6x - 2y = 18 \end{cases}$

$y = 3x + 12$

$6x - 2(3x + 12) = 18$

$-24 = 18$

no solutions

Explain 3 Solving Linear System Models by Substitution

You can use a system of linear equations to model real-world situations.

Example 3 **Solve each real-world situation by using the substitution method.**

(A) Fitness center A has a \$60 enrollment fee and costs \$35 per month. Fitness center B has no enrollment fee and costs \$45 per month. Let t represent the total cost in dollars and m represent the number of months. The system of equations $\begin{cases} t = 60 + 35m \\ t = 45m \end{cases}$ can be used to represent this situation. In how many months will both fitness centers cost the same? What will the cost be?

$60 + 35m = 45m$	Substitute $60 + 35m$ for t in the second equation.
$60 = 10m$	Subtract $35m$ from each side.
$6 = m$	Divide each side by 10.
$t = 45m$	Use one of the original equations.
$= 45(6) = 270$	Substitute 6 for m.
$(6, 270)$	Write the solution as an ordered pair.

Both fitness centers will cost \$270 after 6 months.

(B) High-speed Internet provider A has a \$100 setup fee and costs \$65 per month. High-speed internet provider B has a setup fee of \$30 and costs \$70 per month. Let t represent the total amount paid in dollars and m represent the number of months. The system of equations $\begin{cases} t = 100 + 65m \\ t = 30 + 70m \end{cases}$ can be used to represent this situation. In how many months will both providers cost the same? What will that cost be?

$\boxed{100 + 65m} = 30 + 70m$	Substitute $\boxed{100 + 65m}$ for t in the second equation.
$100 = \boxed{30 + 5m}$	Subtract $\boxed{65}$ m from each side.
$\boxed{70} = \boxed{5}\ m$	Subtract $\boxed{30}$ from each side.
$\boxed{14} = m$	Divide each side by $\boxed{5}$.

DIFFERENTIATE INSTRUCTION

Graphic Organizer

Have students show the steps for solving a system of equations by substitution.

Solving Systems of Equations by Substitution	
Step 1	Solve for one variable in one equation.
Step 2	Substitute the resulting expression into the other equation.
Step 3	Solve that equation to get the value of the other variable.
Step 4	Substitute that value into one of the original equations and solve.
Step 5	Write the values from Steps 3 and 4 in an ordered pair (x, y).
Step 6	Check the solution by substituting into both equations or by graphing.

$t = 30 + 70m$ Use one of the original equations.

$t = 30 + 70(\boxed{14})$ Substitute $\boxed{14}$ for m.

$t = \boxed{1010}$

$(\boxed{14}, \boxed{1010})$ Write the sulotion as an ordered pair.

Both Internet providers will cost \$ 1010 after 14 months.

Reflect

9. If the variables in a real-world situation represent the number of months and cost, why must the values of the variables be greater than or equal to zero?

The values of the variables must be greater than or equal to zero because the total cost and the number of months cannot be negative.

Your Turn

10. A boat travels at a rate of 18 kilometers per hour from its port. A second boat is 34 kilometers behind the first boat when it starts traveling in the same direction at a rate of 22 kilometers per hour to the same port. Let d represent the distance the boats are from the port in kilometers and t represent the amount of time in hours. The system of equations $\begin{cases} d = 18t + 34 \\ d = 22t \end{cases}$ can be used to represent this situation. How many hours will it take for the second boat to catch up to the first boat? How far will the boats be from their port? Use the substitution method to solve this real-world application.

$18t + 34 = 22t$ $\quad d = 22t$

$8.5 = t$ $\quad d = 22(8.5) = 187$

The second boat will catch up in 8.5 hours, and they will be 187 km from their port.

Elaborate

11. When given a system of linear equations, how do you decide which variable to solve for first?

Use a variable that has a coefficient of 1 or −1. If no variables have a coefficient of 1 or −1, look for a variable that will result in the simplest expression.

12. How can you check a solution for a system of equations without graphing?

Substitute the solution into each equation and determine whether all of the equations in the system are true.

13. **Essential Question-Check-In** Explain how you can solve a system of linear equations by substitution.

Solve one equation for one variable and use the result to substitute into the other equation. Solve for the value of the other variable. Then substitute that value into either equation to find the value of the first variable.

ELABORATE

QUESTIONING STRATEGIES

Why can you substitute the value of one variable into either of the original equations to find the value of the other variable? **If there is a solution to the system of equations, the values of the variables will satisfy both equations.**

SUMMARIZE THE LESSON

How do you know if your solution to a system of linear equations is correct? **You can verify your solution by graphing the equations. This allows you to verify that the number of solutions is correct by seeing whether the lines appear to be the same line, two parallel lines, or two lines that intersect at one point. If the lines intersect at one point, you can also substitute the solution back into the original equations to verify your solution.**

LANGUAGE SUPPORT EL

Connect Vocabulary

Remind students that the *substitution method* involves *substituting* an expression from one equation into the other equation. Explain that *to substitute* means *to replace*. Note that the word *substitute* can be used as a noun or an adjective: a substitute (noun) in sports replaces the original player, and a substitute (adjective) teacher replaces the regular teacher. Emphasize that expressions used for substitution in math must always be equal in value to the expression they are replacing.

EVALUATE

ASSIGNMENT GUIDE

Concepts and Skills	Practice
Explore Exploring the Substitution Method of Solving Linear Systems	Exercise 1
Example 1 Solving Consistent, Independent Linear Systems by Substitution	Exercises 2–7, 23–25
Example 2 Solving Special Linear Systems by Substitution	Exercises 8–13, 20
Example 3 Solving Linear System Models by Substitution	Exercises 14–19, 21–22

INTEGRATE MATHEMATICAL PRACTICES

Focus on Reasoning

MP.2 Students can check their solutions for correctness by substituting the values into the original equations and verifying that both solutions make both equations true.

Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

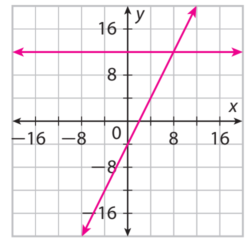
1. In the system of linear equations shown, the value of y is given. Use this value of y to find the value of x and the solution of the system.

$$\begin{cases} y = 12 \\ 2x - y = 4 \end{cases}$$

a. What is the solution of the system?

The solution is $(8, 12)$.

b. Graph the system of linear equations. How do the solutions compare?



The solutions are the same.

Solve each system of linear equations by substitution.

2. $\begin{cases} 5x + y = 8 \\ 2x + y = 5 \end{cases}$

$y = -5x + 8$
$2x + (-5x + 8) = 5$
$-3x + 8 = 5$
$-3x = -3$
$x = 1$
$5(1) + y = 8$
$5 + y = 8$
$y = 3$
The solution is $(1, 3)$.

3. $\begin{cases} x - 3y = 10 \\ x + 5y = -22 \end{cases}$

$x = 3y + 10$
$(3y + 10) + 5y = -22$
$8y + 10 = -22$
$8y = -32$
$y = -4$
$x - 3(-4) = 10$
$x + 12 = 10$
$x = -2$
The solution is $(-2, -4)$.

4. $\begin{cases} 5x - 3y = 22 \\ -4x + y = -19 \end{cases}$

$y = 4x - 19$
$5x - 3(4x - 19) = 22$
$-7x + 57 = 22$
$-7x = -35$
$x = 5$
$-4(5) + y = -19$
$-20 + y = -19$
$y = 1$
The solution is $(5, 1)$.

5. $\begin{cases} x + 7y = -11 \\ -2x - 5y = 4 \end{cases}$

$x = -7y - 11$
$-2(-7y - 11) - 5y = 4$
$9y + 22 = 4$
$9y = -18$
$y = -2$
$x + 7(-2) = -11$
$x - 14 = -11$
$x = 3$
The solution is $(3, -2)$.

6. $\begin{cases} 2x + 6y = 16 \\ 3x - 5y = -18 \end{cases}$

$2x = -6y + 16$
$x = -3y + 8$
$3(-3y + 8) - 5y = -18$
$-14y + 24 = -18$
$-14y = -42$
$y = 3$
$2x + 6(3) = 16$
$2x + 18 = 16$
$2x = -2$
$x = -1$
The solution is $(-1, 3)$.

7. $\begin{cases} 7x + 2y = 24 \\ -6x + 3y = 3 \end{cases}$

$3y = 6x + 3$
$y = 2x + 1$
$7x + 2(2x + 1) = 24$
$11x + 2 = 24$
$11x = 22$
$x = 2$
$-6(2) + 3y = 3$
$-12 + 3y = 3$
$3y = 15$
$y = 5$
The solution is $(2, 5)$.

Exercise	Depth of Knowledge (D.O.K.)	COMMON CORE Mathematical Practices
1	**2** Skills/Concepts	**MP.4** Modeling
2–13	**1** Recall of Information	**MP.2** Reasoning
14–18	**2** Skills/Concepts	**MP.4** Modeling
19	**3** Strategic Thinking	**MP.4** Modeling
20	**2** Skills/Concepts	**MP.2** Reasoning
21	**2** Skills/Concepts	**MP.4** Modeling

Solve each system of linear equations by substitution.

8. $\begin{cases} x + y = 3 \\ -4x - 4y = 12 \end{cases}$

$x = -y + 3$
$-4(-y + 3) - 4y = 12$
$-12 = 12$
There is no solution.

9. $\begin{cases} 3x - 3y = -15 \\ -x + y = 5 \end{cases}$

$y = x + 5$
$3x - 3(x + 5) = -15$
$-15 = -15$
There are infinitely many solutions.

10. $\begin{cases} x - 8y = 17 \\ -3x + 24y = -51 \end{cases}$

$x = 8y + 17$
$-3(8y + 17) + 24y = -51$
$-51 = -51$
There are infinitely many solutions.

11. $\begin{cases} 5x - y = 18 \\ 10x - 2y = 32 \end{cases}$

$-y = -5x + 18$
$y = 5x - 18$
$10x - 2(5x - 18) = 32$
$36 = 32$
There is no solution.

12. $\begin{cases} -2x - 3y = 12 \\ -4x - 6y = 24 \end{cases}$

$x = -\frac{3}{2}y - 6$
$-4\left(-\frac{3}{2}y - 6\right) - 6y = 24$
$24 = 24$
There are infinitely many solutions.

13. $\begin{cases} 3x + 4y = 36 \\ 6x + 8y = 48 \end{cases}$

$3x = -4y + 36$
$x = -\frac{4}{3}y + 12$
$6\left(-\frac{4}{3}y + 12\right) + 8y = 48$
$72 = 48$
There is no solution.

Solve each real-world situation by using the substitution method.

14. The number of DVDs sold at a store in a month was 920 and the number of DVDs sold decreased by 12 per month. The number of Blu-ray discs sold in the same store in the same month was 502 and the number of Blu-ray discs sold increased by 26 per month. Let d represent the number of discs sold and t represent the time in months.

The system of equations $\begin{cases} d = 920 - 12t \\ d = 502 + 26t \end{cases}$ can be used to represent this situation. If this trend continues, in how many months will the number of DVDs sold equal the number of Blu-ray discs sold? How many of each is sold in that month?

$920 - 12t = 502 + 26t$
$418 = 38t$
$11 = t$

$d = 502 + 26t$
$d = 502 + 26(11)$
$d = 788$

There will be 788 DVDs and 788 Blu-Ray discs sold per month in 11 months.

15. One smartphone plan costs \$30 per month for talk and messaging and \$8 per gigabyte of data used each month. A second smartphone plan costs \$60 per month for talk and messaging and \$3 per gigabyte of data used each month. Let c represent the total cost in dollars and d represent the amount of data used in gigabytes. The system of equations $\begin{cases} c = 30 + 8d \\ c = 60 + 3d \end{cases}$ can be used to represent this situation. How many gigabytes would have to be used for the plans to cost the same? What would that cost be?

$30 + 8d = 60 + 3d$
$5d = 30$
$d = 6$

$c = 30 + 8(6)$
$c = 78$

Both plans would cost \$78 if 6 gigabytes of data are used.

MODELING

Some students may have difficulty using the substitution method. Suggest to them that they graph the system first, and then use the graph to guide and check their work as they use substitution.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Reasoning

MP.2 Remind students that when using the substitution method to solve a system, it does not matter which variable you solve for first. Demonstrate that whether you solve for x first or y first, you will obtain the same solution. Therefore, you can choose to solve in whichever order is easier. If possible, solve for the variable that has a coefficient of 1 or −1.

Exercise	Depth of Knowledge (D.O.K.)	COMMON CORE Mathematical Practices
22	**3** Strategic Thinking H.O.T.	**MP.4** Modeling
23	**3** Strategic Thinking H.O.T.	**MP.6** Precision
24–25	**3** Strategic Thinking H.O.T.	**MP.3** Logic

AVOID COMMON ERRORS

Students often think they have solved a system of equations after finding the value of only one variable. Remind them that the solution is an ordered pair.

16. A movie theater sells popcorn and fountain drinks. Brett buys 1 popcorn bucket and 3 fountain drinks for his family, and pays a total of \$9.50. Sarah buys 3 popcorn buckets and 4 fountain drinks for her family, and pays a total of \$19.75. If p represents the number of popcorn buckets and d represents the number of drinks, then the system of equations $\begin{cases} 9.50 = p + 3d \\ 19.75 = 3p + 4d \end{cases}$ can be used to represent this situation. Find the cost of a popcorn bucket and the cost of a fountain drink.

$9.50 - 3d = p$ $\qquad$ $9.50 = p + 3(1.75)$

$19.75 = 3(9.50 - 3d) + 4d$ $\qquad$ $9.50 = p + 5.25$

$1.75 = d$ $\qquad$ $4.25 = p$

The cost of a bucket of popcorn is \$4.25 and the cost of a fountain soda is \$1.75.

17. Jen is riding her bicycle on a trail at the rate of 0.3 kilometer per minute. Michelle is 11.2 kilometers behind Jen when she starts traveling on the same trail at a rate of 0.44 kilometer per minute. Let d represent the distance in kilometers the bicyclists are from the start of the trail and t represent the time in minutes. The system of equations $\begin{cases} d = 0.3t + 11.2 \\ d = 0.44t \end{cases}$ can be used to represent this situation. How many minutes will it take Michelle to catch up to Jen? How far will they be from the start of the trail? Use the substitution method to solve this real-world application.

$0.3t + 11.2 = 0.44t$ $\qquad$ $d = 0.44t$

$80 = t$ $\qquad$ $= 0.44(80) = 35.2$

Michelle will catch up in 80 minutes, and they will be 35.2 km from the start.

18. **Geometry** The length of a rectangular room is 5 feet more than its width. The perimeter of the room is 66 feet. Let L represent the length of the room and W represent the width in feet. The system of equations $\begin{cases} L = W + 5 \\ 66 = 2L + 2W \end{cases}$ can be used to represent this situation. What are the room's dimensions?

$66 = 2(W + 5) + 2W$ $\qquad$ $L = W + 5$

$56 = 4W$ $\qquad$ $L = 14 + 5$

$14 = W$ $\qquad$ $L = 19$

The room has a width of 14 feet and a length of 19 feet.

19. A cable television provider has a \$55 setup fee and charges \$82 per month, while a satellite television provider has a \$160 setup fee and charges \$67 per month. Let c represent the total cost in dollars and t represent the amount of time in months. The system of equations $\begin{cases} c = 55 + 82t \\ c = 160 + 67t \end{cases}$ can be used to represent this situation.

a. In how many months will both providers cost the same? What will that cost be?

$55 + 82t = 160 + 67t$ $\qquad$ $c = 55 + 82t$

$15t = 105$ $\qquad$ $c = 55 + 82(7)$

$t = 7$ $\qquad$ $= 629$

Both providers will cost \$629 in 7 months.

b. If you plan to move in 12 months, which provider would be less expensive? Explain.

Satellite would be less expensive because it costs less per month than cable and 12 months is after 7 months.

20. Determine whether each of the following systems of equations have one solution, infinitely many solutions, or no solution. Select the correct answer for each lettered part.

a. $\begin{cases} x + y = 5 \\ -6y - 6y = 30 \end{cases}$ **none**

b. $\begin{cases} x + y = 7 \\ 5x + 2y = 23 \end{cases}$ **one**

c. $\begin{cases} 3x + y = 5 \\ 6x + 2y = 12 \end{cases}$ **none**

d. $\begin{cases} 2x + 5y = -12 \\ x + 7y = -15 \end{cases}$ **one**

e. $\begin{cases} 3x + 5y = 17 \\ -6x - 10y = -34 \end{cases}$ **infinitely many**

21. **Finance** Adrienne invested a total of $1900 in two simple-interest money market accounts. Account A paid 3% annual interest and account B paid 5% annual interest. The total amount of interest she earned after one year was $83. If a represents the amount invested in dollars in account A and b represents the amount invested in dollars in account B, the system of equations $\begin{cases} a + b = 1900 \\ 0.03a + 0.05b = 83 \end{cases}$ can represent this situation. How much did Adrienne invest in each account?

$$a = -b + 1900$$
$$0.03(-b + 1900) + 0.05b = 83$$
$$0.02b = 26$$
$$b = 1300$$

$$a + b = 1900$$
$$a + (1300) = 1900$$
$$a = 600$$

Adrienne invested $600 in account A and $1300 in account B.

H.O.T. Focus on Higher Order Thinking

22. **Real-World Application** The Sullivans are deciding between two landscaping companies. Evergreen charges a $79 startup fee and $39 per month. Eco Solutions charges a $25 startup fee and $45 per month. Let c represent the total cost in dollars and t represent the time in months. The system of equations $\begin{cases} c = 39t + 79 \\ c = 45t + 25 \end{cases}$ can be used to represent this situation.

a. In how many months will both landscaping services cost the same? What will that cost be?

$$39t + 79 = 45t + 25$$
$$54 = 6t$$
$$9 = t$$

$$c = 45t + 79$$
$$c = 39(9) + 79$$
$$= 430$$

Both will cost $430 in 9 months.

b. Which landscaping service will be less expensive in the long term? Explain.

Evergreen will be less expensive than Eco Solutions in the long term. They will cost the same after 9 months but the rate of change for Evergreen is less than the rate of change for Eco Solutions.

VISUAL CUES

After isolating one variable in one equation, some students may find it helpful to highlight the variable with a colored pencil, and then highlight the same variable in the other equation. This will help them remember where in the other equation to substitute the expression for that variable.

JOURNAL

Have students write a journal entry that summarizes how to solve a system of equations by substitution. Students should mention how to decide which equation to use for the substitution.

23. **Multiple Representations** For the first equation in the system of linear equations below, write an equivalent equation without denominators. Then solve the system.

$$\begin{cases} \frac{x}{5} + \frac{y}{3} = 6 \\ x - 2y = 8 \end{cases}$$

$15\left(\frac{x}{5} + \frac{y}{3}\right) = 6$

$3x + 5y = 90$

$x - 2y = 8$

$x = 2y + 8$

$3(2y + 8) + 5y = 90$

$6y + 24 + 5y = 90$

$11y + 24 = 90$

$11y = 66$

$y = 6$

$x - 2(6) = 8$

$x - 12 = 8$

$x = 20$

The solution is $(20, 6)$.

24. **Conjecture** Is it possible for a system of three linear equations to have one solution? If so, give an example.

Yes; the solution is an ordered pair that is a solution of each of the equations. For example, the solution of the system containing the equations $3x - y = 5$, $x + y = 3$, and $x = 2y$ is $(2, 1)$.

25. **Conjecture** Is it possible to use substitution to solve a system of linear equations if one equation represents a horizontal line and the other equation represents a vertical line? Explain.

No, the equation of a horizontal line is in the form $y = a$ and the equation of a vertical line is in the form $x = b$. The horizontal line equation has no x-term and the vertical line equation has no y-term.

Lesson Performance Task

A company breaks even from the production and sale of a product if the total revenue equals the total cost. Suppose an electronics company is considering producing two types of smartphones. To produce smartphone A, the initial cost is \$20,000 and each phone costs \$150 to produce. The company will sell smartphone A at \$200. Let $C(a)$ represent the total cost in dollars of producing a units of smartphone A. Let $R(a)$ represent the total revenue, or money the company takes in due to selling a units of smartphone A. The system of equations $\begin{cases} C(a) = 20{,}000 + 150a \\ R(a) = 200a \end{cases}$ can be used to represent the situation for phone A.

To produce smartphone B, the initial cost is \$44,000 and each phone costs \$200 to produce. The company will sell smartphone B at \$280. Let $C(b)$ represent the total cost in dollars of producing b units of smartphone B and $R(b)$ represent the total revenue from selling b units of smartphone B. The system of equations $\begin{cases} C(b) = 44{,}000 + 200b \\ R(b) = 280b \end{cases}$ can be used to represent the situation for phone B.

Solve each system of equations and interpret the solutions. Then determine whether the company should invest in producing smartphone A or smartphone B. Justify your answer.

Smartphone A:

$200a = 20{,}000 + 150a$

$50a = 20{,}000$

$a = 400$

$R(a) = 200a$

$= 200(400)$

$= 80{,}000$

The company will break even selling 400 units of smartphone A for a total $80,000.

Smartphone B:

$280b = 44{,}000 + 200b$

$80b = 44{,}000$

$b = 550$

$R(b) = 280b$

$= 280(550)$

$= 154{,}000$

The company will break even selling 550 units of smartphone B for a total of $154,000.

Some students may say that the company should invest in producing smartphone A because the initial cost of producing smartphone A is less than that of producing smartphone B and fewer units of smartphone A would need to be sold for the company to break even. Other students may argue that the company should consider other factors, such as increasing the sale price of smartphone B or looking for ways to cut the initial cost of production.

EXTENSION ACTIVITY

Many companies sell accessories for smartphones. Have students research the different types of accessories sold and make conjectures about how a company might use a system of equations to find the break-even cost in selling these accessories.

QUESTIONING STRATEGIES

How is profit determined? **Profit = total revenue − total cost = $R - C$**

When total revenue equals total cost, what is the profit? What is this situation called? **The profit is \$0; this is called the break-even point. The break-even points in the Lesson Performance Task are when $C(a) = R(a)$ for smartphone A and $C(b) = R(b)$ for smartphone B.**

INTEGRATE MATHEMATICAL PRACTICES

Focus on Technology

MP.8 To check their solutions for smartphone A, have students use graphing calculators to graph $y = 20{,}000 + 150x$ and $y = 200x$ on the same coordinate plane. Then they can go to the **CALC** menu and select the intersect feature to find the coordinates of the point of intersection. Students can use the same procedure with the equations $y = 44{,}000 + 200x$ and $y = 280x$ to check their answers for smartphone B.

Scoring Rubric

2 points: Student correctly solves the problem and explains his/her reasoning.

1 point: Student shows good understanding of the problem but does not fully solve or explain his/her reasoning.

0 points: Student does not demonstrate understanding of the problem.