LESSON 11.1

Solving Linear Systems by Graphing

Common Core Math Standards

The student is expected to:

COMMON CORE **A-REI.C.6**

Solve systems of linear equations... approximately (e.g., with graphs) . . .

Mathematical Practices

COMMON CORE **MP.5 Using Tools**

Language Objective

Use graphs to explain the difference between systems of equations that are inconsistent, consistent and dependent, and consistent and independent.

ENGAGE

Essential Question: How can you find the solution of a system of linear equations by graphing?

Graph the lines. If the graphs intersect in one point (a, b), the system has one solution, (a, b). If the two lines do not intersect, the system has no solution. If the graphs coincide, that is, if they are the same line, the system has infinitely many solutions.

PREVIEW: LESSON PERFORMANCE TASK

View the Engage section online. Discuss how the speed of the current in a river can affect the speed of a boat going upstream or downstream. Then preview the Lesson Performance Task.

Name ____________ Class ________ Date ________

11.1 Solving Linear Systems by Graphing

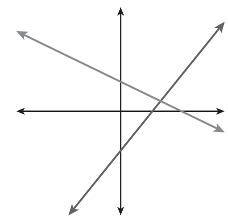
Essential Question: How can you find the solution of a system of linear equations by graphing?

Resource Locker

Explore Types of Systems of Linear Equations

A **system of linear equations,** also called a *linear system,* consists of two or more linear equations that have the same variables. A **solution of a system of linear equations** with two variables is any ordered pair that satisfies all of the equations in the system.

Graph A



(A) Describe the relationship between the two lines in Graph A.

The two lines have different slopes and they intersect at exactly one point.

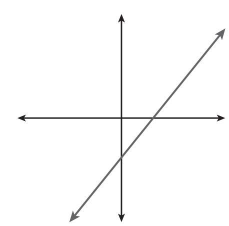
(B) What do you know about every point on the graph on a linear equation?

Every point on the line is a solution of the linear equation.

(C) How many solutions does a system of two equations have if the graphs of the two equations intersect at exactly one point?

exactly one solution

Graph B

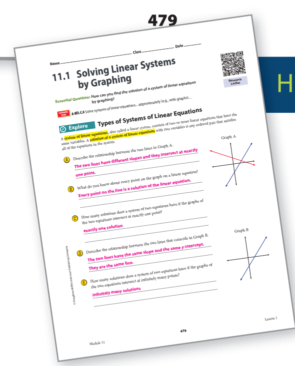


(D) Describe the relationship between the two lines that coincide in Graph B.

The two lines have the same slope and the same *y*-intercept. They are the same line.

(E) How many solutions does a system of two equations have if the graphs of the two equations intersect at infinitely many points?

infinitely many solutions



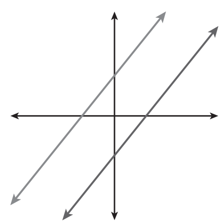
HARDCOVER

Turn to Lesson 11.1 in the hardcover edition.

Ⓕ Describe the relationship between the two lines in Graph C.

The two lines have the same slope and different *y*-intercepts. They are parallel lines.

Graph C



Ⓖ How many solutions does a system of two equations have if the graphs of the two equations do not intersect?

no solutions

Reflect

1. **Discussion** Explain why the solution of a system of two equations is represented by any point where the two graphs intersect.
   All points on the graphs of each equation represent the solutions to those equations. If any point is on both graphs, then it is a solution of both equations.

Explain 1 Solving Consistent, Independent Linear Systems by Graphing

A **consistent system** is a system with at least one solution. Consistent systems can be either independent or dependent.

An **independent system** has exactly one solution. The graph of an independent system consists of two lines that intersect at exactly one point. A **dependent system** has infinitely many solutions. The graph of a dependent system consists of two coincident lines, or the same line.

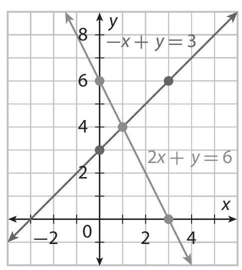
A system that has no solution is an **inconsistent system**.

Example 1 **Solve the system of linear equations by graphing. Check your answer.**

Ⓐ $\begin{cases} 2x + y = 6 \\ -x + y = 3 \end{cases}$

Find the intercepts for each equation, plus a third point for a check. Then graph.

$2x + y = 6$	$-x + y = 3$
x-intercept: 3	x-intercept: −3
y-intercept: 6	y-intercept: 3
third point: $(-1, 8)$	third point: $(3, 6)$



The two lines appear to intersect at $(1, 4)$. Check.

$2x + y = 6 \qquad -x + y = 3$

$2(1) + 4 \stackrel{?}{=} 6 \qquad -(1) + 4 \stackrel{?}{=} 3$

$6 = 6 \qquad 3 = 3$

The point satisfies both equations, so the solution is $(1, 4)$.

PROFESSIONAL DEVELOPMENT

Integrate Mathematical Practices

This lesson provides an opportunity to address Mathematical Practice **MP.5**, which calls for students to “use tools.” To get a correct solution by graphing, students must use graph paper and a straightedge to accurately draw both lines in the system. Using a hand-drawn grid or trying to graph the lines without a guide to make them straight will lead to errors. Students can also use a graphing calculator to solve systems by graphing.

EXPLORE

Types of Systems of Linear Equations

INTEGRATE TECHNOLOGY

Students have the option of completing the Explore activity either in the book or online.

QUESTIONING STRATEGIES

What is the difference between the solution of a system of two linear equations and the solution of an equation in one variable? **The solution of a system of equations is an ordered pair that makes both equations true. The solution of an equation in one variable is a single number that makes the equation true.**

EXPLAIN 1

Solving Consistent, Independent Linear Systems by Graphing

CONNECT VOCABULARY EL

Have students consider the word *consistent*. One definition of *consistent* is *compatible or in agreement*. Systems that are consistent are in agreement somewhere, that is, they have a solution. *Inconsistent* is the opposite of *consistent*; it means *not in agreement*. Systems that are inconsistent are not in agreement and have no solution.

QUESTIONING STRATEGIES

Why is it not sufficient to check your proposed solution in just one of the equations? **There are infinitely many ordered pairs that will satisfy one equation but not the other. Only one ordered pair will satisfy both equations.**

CONNECT VOCABULARY EL

Point out that the terms *dependent* and *independent* have a somewhat different meaning when used to describe systems of equations than when used to describe dependent and independent variables. However, both uses rely on the same underlying meanings of the words. In a dependent system, the two lines are the same, so you could say that each one depends on the other. In an independent system, the two lines go their separate ways and do not depend on one another.

Ⓑ $\begin{cases} y = 2x - 2 \\ 3y + 6x = 18 \end{cases}$

Find the intercepts for each equation, plus a third point for a check. Then graph.

$y = 2x - 2$

x-intercept: **1**

y-intercept: **−2**

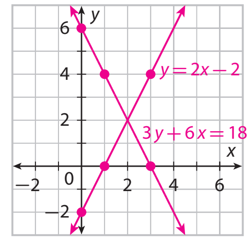
third point: $(3,$ **4**$)$

$3y + 6x = 18$

x-intercept: **3**

y-intercept: **6**

third point: $(1,$ **4**$)$



The two lines appear to intersect at **(2, 2)**. Check.

$y = 2x - 2$

2 $\stackrel{?}{=} 2($**2**$) - 2$

2 $=$ **2**

$y + 2x = 6$

2 $+ 2($**2**$) \stackrel{?}{=} 6$

6 $= 6$

The point satisfies both equations, so the solution is **(2, 2)**.

Reflect

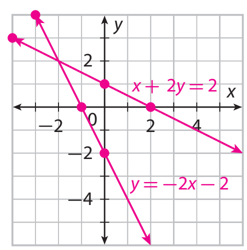
2. How do you know that the systems of equations are consistent? How do you know that they are independent?

I know the systems of equations are consistent because they have at least one solution and they are independent because they have exactly one solution.

Your Turn

Solve the system of linear equations by graphing. Check your answer.

3. $\begin{cases} y = -2x - 2 \\ x + 2y = 2 \end{cases}$



The two lines appear to intersect at (−2, 2).

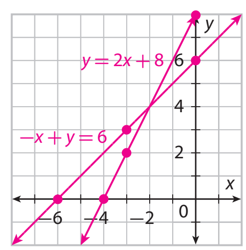
$y = -2x - 2$ $\quad$ $x + 2y = 2$

$2 \stackrel{?}{=} -2(-2) - 2$ $\quad$ $-2 + 2(2) \stackrel{?}{=} 2$

$2 = 2$ $\quad$ $2 = 2$

The solution is (−2, 2)

4. $\begin{cases} y = 2x + 8 \\ -x + y = 6 \end{cases}$



The two lines appear to intersect at (−2, 4).

$y = 2x + 8$ $\quad$ $-x + y = 6$

$4 \stackrel{?}{=} 2(-2) + 8$ $\quad$ $-(-2) + 4 \stackrel{?}{=} 6$

$4 = 4$ $\quad$ $6 = 6$

The solution is (−2, 4).

COLLABORATIVE LEARNING

Peer-to-Peer Activity

Have students work in pairs to find an approximate solution to a system of linear equations whose solution has non-integer coordinates. One should estimate the solution by graphing, and the other should check the reasonableness of the estimate. Have each pair of students solve several systems of equations, taking turns in the two roles.

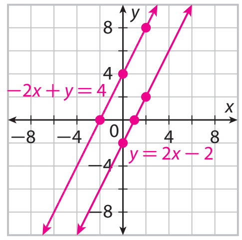
Explain 2 Solving Special Linear Systems by Graphing

Example 2 **Solve the special system of equations by graphing and identify the system.**

Ⓐ $\begin{cases} y = 2x - 2 \\ -2x + y = 4 \end{cases}$

Find the intercepts for each equation, plus a third point for a check.

$y = 2x - 2$	$-2x + y = 4$
x-intercept: 1	x-intercept: -2
y-intercept: -2	y-intercept: 4
third point: (2, 2)	third point: (2, 8)

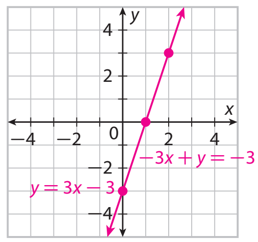


The two lines don't intersect, so there is no solution.
The two lines have the same slope and different y-intercepts so they will never intersect. This is an inconsistent system.

Ⓑ $\begin{cases} y = 3x - 3 \\ -3x + y = -3 \end{cases}$

Find the intercepts for each equation, plus a third point for a check.

$y = 3x - 3$	$-3x + y = -3$
x-intercept: **1**	x-intercept: **1**
y-intercept: **−3**	y-intercept: **−3**
third point: (2, **3**)	third point: (2, **3**)



The two lines coincide, so there are **infinitely many** solutions.

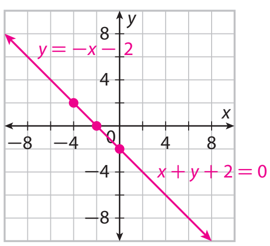
They have the same slope and y-intercept; therefore, they are **the same** line(s) / equation(s).

This is a **consistent** and **dependent** system.

Your Turn

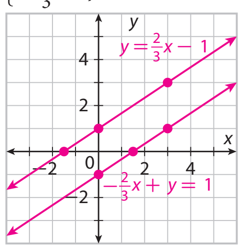
Solve the special system of linear equations by graphing. Check your answer.

5. $\begin{cases} y = -x - 2 \\ x + y + 2 = 0 \end{cases}$



The two lines coincide, so there are infinitely many solutions.

6. $\begin{cases} y = \frac{2}{3}x - 1 \\ -\frac{2}{3}x + y = 1 \end{cases}$



The two lines don't intersect, so there is no solution.

EXPLAIN 2

Solving Special Linear Systems by Graphing

QUESTIONING STRATEGIES

If you noticed that two lines had the same slope, what conclusion would you reach about the system of equations? Explain. **The lines are either the same or they are parallel. If they have the same y-intercept, they are the same line (forming a consistent and dependent system). If they do not have the same y-intercept, they are parallel (forming an inconsistent system).**

INTEGRATE MATHEMATICAL PRACTICES

Focus on Critical Thinking

MP.3 Have students graph three linear equations on a grid with the following conditions: two of the lines represent an inconsistent system of the equations, and two of the lines represent a consistent and independent system of equations. Students should recognize that two of the lines must be parallel and the third line must intersect the other two.

DIFFERENTIATE INSTRUCTION

Graphic Organizer

Have students complete the graphic organizer that shows the possible solutions to systems of equations.

Graphs of Equations	Number of Solutions	Type of System
Intersecting lines	**1**	**Consistent and independent**
Same line	**Infinite number**	**Consistent and dependent**
Parallel lines	**0**	**Inconsistent**

EXPLAIN 3

Estimating Solutions of Linear Systems by Graphing

AVOID COMMON ERRORS

Make sure students understand that a graph can give only an approximate solution, so they must verify the solution using algebra.

QUESTIONING STRATEGIES

Why might an estimated solution be very far from the actual solution? Explain. **It could be far from it if the estimations of the *x*- and *y*-values were not very precise. That's why it is important to be as precise as possible.**

If the solution is estimated very precisely, how does this affect your solution check? **The more precisely the solution is estimated, the closer the equations should be to being true.**

AVOID COMMON ERRORS

Some students may reverse the *x*- and *y*-coordinates in their solutions. Emphasize that checking their solutions will help correct such errors.

INTEGRATE TECHNOLOGY

Have students use the intersect feature of a graphing calculator to approximate a solution to a system of linear equations.

Explain 3 Estimating Solutions of Linear Systems by Graphing

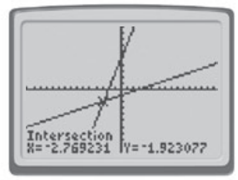
You can estimate the solution of a linear system of equations by graphing the system and finding the approximate coordinates of the intersection point.

Example 3 Estimate the solution of the linear system by graphing.

Ⓐ $\begin{cases} x - 3y = 3 \\ -5x + 2y = 10 \end{cases}$

Graph the equations using a graphing calculator.

Y1 = (3 − X)/(−3) and Y2 = (10 + 5X)/2

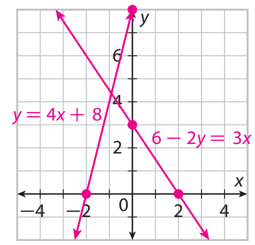


Find the point of intersection.

The two lines appear to intersect at about (−2.8, −1.9).

Ⓑ $\begin{cases} 6 - 2y = 3x \\ y = 4x + 8 \end{cases}$

Graph each equation by finding intercepts.



The two lines appear to intersect at about **(−1, 4.5)**.

Check to see if **(−1, 4.5)** makes both equations true.

$6 - 2y = 3x$ $\quad\quad$ $y = 4x + 8$

$6 - 2(\mathbf{4.5}) \stackrel{?}{=} 3(\mathbf{-1})$ $\quad\quad$ $\mathbf{4.5} \stackrel{?}{=} 4(\mathbf{-1}) + 8$

$\mathbf{-3} \approx \mathbf{-3}$ $\quad\quad$ $\mathbf{4.5} \approx \mathbf{4}$

The point does not satisfy both equations, but the results are close.

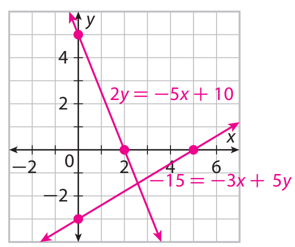
So, **(−1, 4.5)** is an approximate solution.

Your Turn

Estimate the solution of the linear system of equations by graphing.

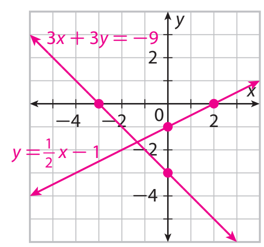
7. $\begin{cases} 2y = -5x + 10 \\ -15 = -3x + 5y \end{cases}$

about (2.5, −1.5)



8. $\begin{cases} 3x + 3y = -9 \\ y = \frac{1}{2}x - 1 \end{cases}$

about (−1.5, −1.5)



LANGUAGE SUPPORT EL

Connect Vocabulary

Explain that the prefix *in-* means not, so *independent* is the opposite of *dependent* and *inconsistent* is the opposite of *consistent.* Ask students what other prefixes they have seen turn a word into its opposite. They may think of *un-* as in *unhappy*; *im-* as in *impossible;* and *non-* as in *nonfiction.* Have them list several words and their meanings as examples of how these prefixes function.

Explain 4 Interpreting Graphs of Linear Systems to Solve Problems

You can solve problems with real-world context by graphing the equations that model the problem and finding a common point.

Example 4 **Rock and Bowl charges \$2.75 per game plus \$3 for shoe rental. Super Bowling charges \$2.25 per game and \$3.50 for shoe rental. For how many games will the cost to bowl be approximately the same at both places? What is that cost?**

Analyze Information

Identify the important information.

- Rock and Bowl charges \$ **2.75** per game plus \$ **3** for shoe rental.
- Super Bowling charges \$ **2.25** per game and \$ **3.50** for shoe rental.
- The answer is the number of games played for which the total cost is approximately the same at both bowling alleys.

Formulate a Plan

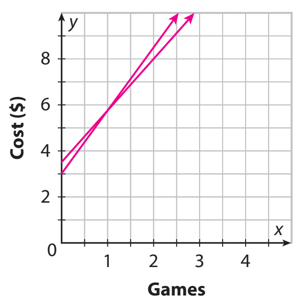
Write a system of linear equations, where each equation represents the price at each bowling alley.

$$\begin{cases} y = 2.75x + 3 \\ y = 2.25x + 3.50 \end{cases}$$

Solve

Graph $y = 2.75x + 3$ and $y = 2.25x + 3.50$.

The lines appear to intersect at **(1, 5.75)**. So, the cost at both places will be the same for **1** game(s) bowled and that cost will be **\$5.75**.



Justify and Evaluate

Check **(1, 5.75)** using both equations.

$2.75(\mathbf{1}) + 3 = \mathbf{5.75}$ $2.25(\mathbf{1}) + 3.5 = \mathbf{5.75}$

Reflect

9. Which bowling alley costs more if you bowl more than 1 game? Explain how you can tell by looking at the graph.
The Rock and Bowl costs more, because after 1 on the graph, the line that represents the Rock and Bowl is higher than the line for Super Bowling.

EXPLAIN 4

Interpreting Graphs of Linear Systems to Solve Problems

INTEGRATE MATHEMATICAL PRACTICES

Focus on Math Connections

MP.1 Point out to students that finding solutions to linear systems of equations is important in the real world. For example, businesses can use systems of equations to model income and expenses and predict profits.

QUESTIONING STRATEGIES

When you graph a system of linear equations that represents a real-world problem, why does the intersection of the two lines represent the solution to the problem? **Every point on a line satisfies the related linear equation. A point that is on both lines (the intersection point) satisfies both equations, so it represents the solution to the problem.**

AVOID COMMON ERRORS

Students sometimes have difficulty correctly assigning x and y variables in real-world problems. Remind them that the value of y cannot be determined unless the value of x is known. In other words, y is dependent on x.

ELABORATE

INTEGRATE MATHEMATICAL PRACTICES

Focus on Modeling

MP.4 When solving a system of equations by graphing, students should recognize that they can adjust the scale of a graph, setting each grid square to represent a smaller unit, in order to estimate a solution more precisely. The intersection of the lines represents an exact rather than an approximate solution when a solution check reveals that it satisfies both equations.

SUMMARIZE THE LESSON

How can you use a graph to solve a system of linear equations? **First, graph both equations in the system. If the lines are parallel, there is no solution. If the lines are the same, there are infinitely many solutions. If the lines intersect in one point, the coordinates of that point are the solution. If the intersection is on the intersection of grid lines, you can find an exact solution. If not, you can estimate the solution.**

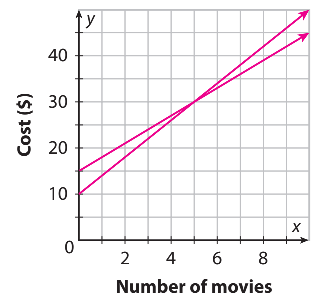
Your Turn

10. Video club A charges $10 for membership and $4 per movie rental. Video club B charges $15 for membership and $3 per movie rental. For how many movie rentals will the cost be the same at both video clubs? What is that cost? Write a system and solve by graphing.

$$\begin{cases} y = 4x + 10 \\ y = 3x + 15 \end{cases}$$

The lines appear to intersect at (5, 30).

The cost will be the same for renting 5 movies, and that cost will be $30.



Elaborate

11. When a system of linear equations is graphed, how is the graph of each equation related to the solutions of that equation?

The solution is indicated by the point of intersection.

12. **Essential Question Check-In** How does graphing help you solve a system of linear equations?

If the lines intersect at one point, the solution is the coordinates of the point of intersection. If the lines do not intersect at one point, the system has either infinitely many solutions or no solutions.

Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

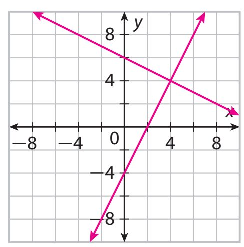
1. Is the following statement correct? Explain.

A system of two equations has no solution if the graphs of the two equations are coincident lines.

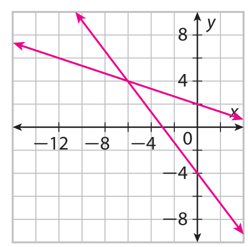
No, if the graphs of the two equations are coincident lines, the system has infinitely many solutions.

Solve the system of linear equations by graphing. Check your answer.

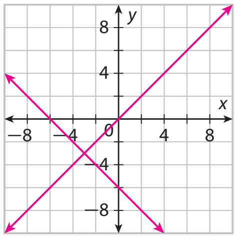
2. $\begin{cases} y = 2x - 4 \\ x + 2y = 12 \end{cases}$ **(4, 4)**



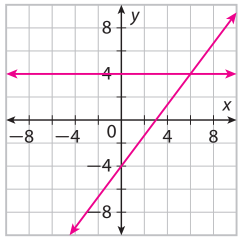
3. $\begin{cases} y = -\frac{1}{3}x + 2 \\ y + 4 = -\frac{4}{3}x \end{cases}$ **(−6, 4)**



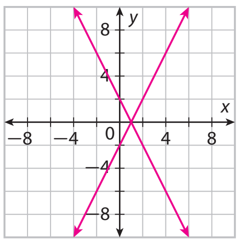
4. $\begin{cases} y = -x - 6 \\ y = x \end{cases}$ **(−3, −3)**



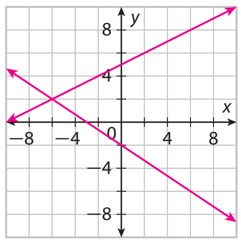
5. $\begin{cases} y = \frac{4}{3}x - 4 \\ y = 4 \end{cases}$ **(6, 4)**



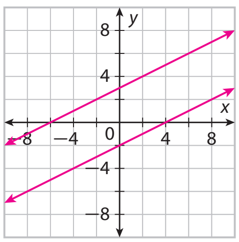
6. $\begin{cases} y = -2x + 2 \\ y + 2 = 2x \end{cases}$ **(1, 0)**



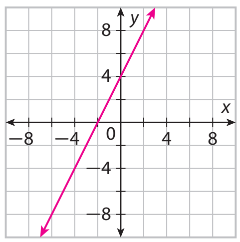
7. $\begin{cases} y = \frac{1}{2}x + 5 \\ \frac{2}{3}x + y = -2 \end{cases}$ **(−6, 2)**



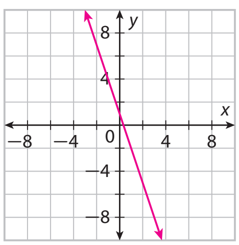
8. $\begin{cases} y = \frac{1}{2}x - 2 \\ -\frac{1}{2}x + y - 3 = 0 \end{cases}$ **No solutions**



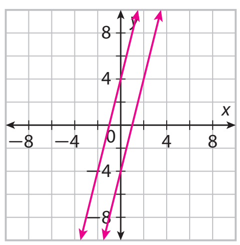
9. $\begin{cases} y = 2x + 4 \\ -4x + 2y = 8 \end{cases}$ **Infinitely many solutions**



10. $\begin{cases} y = -3x + 1 \\ 12x + 4y = 4 \end{cases}$ **Infinitely many solutions**



11. $\begin{cases} y = 4x + 4 \\ -4x + y + 4 = 0 \end{cases}$ **No solutions**



EVALUATE

Personal Math Trainer

ASSIGNMENT GUIDE

Concepts and Skills	Practice
Explore Types of Systems of Linear Equations	Exercises 1, 22
Example 1 Solving Consistent, Independent Linear Systems by Graphing	Exercises 2–7, 24
Example 2 Solving Special Linear Systems by Graphing	Exercises 8–13, 23
Example 3 Estimating Solutions of Linear Systems by Graphing	Exercises 14–17
Example 4 Interpreting Graphs of Linear Systems to Solve Problems	Exercises 18–21, 25

MULTIPLE REPRESENTATIONS

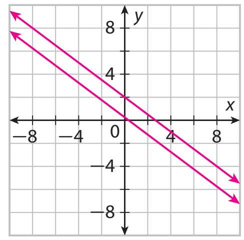
Students can choose one of several methods to graph the linear equations in the systems. They can make a table of values and find ordered pairs; they can find the *x*- and *y*-intercepts for each line; or they can find the slope and *y*-intercept of each line.

Exercise	Depth of Knowledge (D.O.K.)	COMMON CORE Mathematical Practices
1	**1** Recall of Information	**MP.2** Reasoning
2–13	**1** Recall of Information	**MP.4** Modeling
14–17	**2** Skills/Concepts	**MP.4** Modeling
18–21	**2** Skills/Concepts	**MP.4** Modeling
22	**2** Skills/Concepts	**MP.2** Reasoning
23–24	**3** Strategic Thinking H.O.T.	**MP.2** Reasoning
25	**3** Strategic Thinking H.O.T.	**MP.4** Modeling

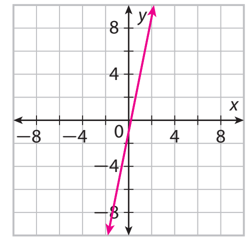
MULTIPLE REPRESENTATIONS

Some students may need help graphing linear systems. Remind students that they can solve the equation for y to give the slope-intercept form of the equation. They can then plot the point for the y-intercept, use the slope to find another point, and then draw a line through the points.

12. $\begin{cases} y = -\frac{3}{4}x + \frac{1}{4} \\ \frac{3}{4}x + y - 2 = 0 \end{cases}$ **No solutions**

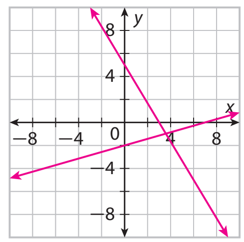


13. $\begin{cases} y = 5x - 1 \\ -5x + y + 4 = 3 \end{cases}$ **Infinitely many solutions**

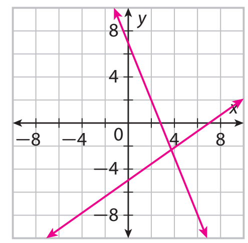


Estimate the solution of the linear system of equations by graphing.

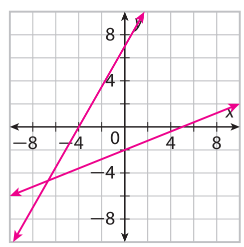
14. $\begin{cases} 3y = -5x + 15 \\ -14 = -2x + 7y \end{cases}$ **approximately (4, −1)**



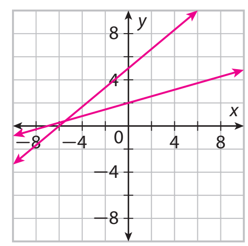
15. $\begin{cases} 2y + 5x = 14 \\ -35 = -5x + 7y \end{cases}$ **approximately (4, −2)**



16. $\begin{cases} \frac{4}{7}y = x + 4 \\ 2x - 5y = 10 \end{cases}$ **approximately (−7, −5)**



17. $\begin{cases} 6y = 5x + 30 \\ 2 = -\frac{2}{7}x + y \end{cases}$ **approximately (−6, 0)**

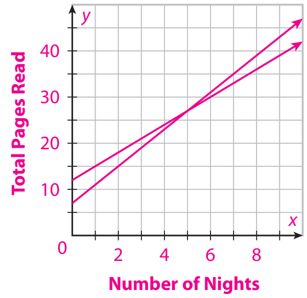


Solve by graphing. Give an approximate solution if necessary.

18. Wren and Jenni are reading the same book. Wren is on page 12 and reads 3 pages every night. Jenni is on page 7 and reads 4 pages every night. After how many nights will they have read the same number of pages? How many pages will that be?

$\begin{cases} y = 3x + 12 \\ y = 4x + 7 \end{cases}$ **(5, 27)**

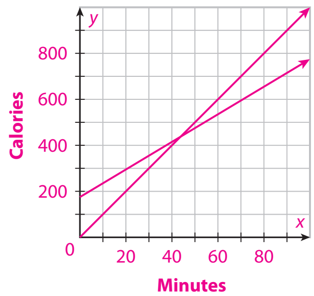
Wren and Jenni will each have read 27 pages after 5 nights.



19. Rusty burns 6 calories per minute swimming and 10 calories per minute jogging. In the morning, Rusty burns 175 calories walking and swims for *x* minutes. In the afternoon, Rusty will jog for *x* minutes. How many minutes must he jog to burn at least as many calories *y* in the afternoon as he did in the morning? Round your answer up to the next whole number of minutes.

$\begin{cases} y = 6x + 175 \\ y = 10x \end{cases}$ **approximately (45, 450)**

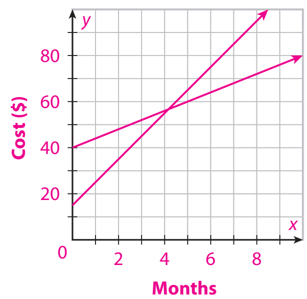
After about 45 minutes, Rusty will have burned about 450 calories each time.



20. A gym membership at one gym costs $10 every month plus a one-time membership fee of $15, and a gym membership at another gym costs $4 every month plus a one-time $40 membership fee. After about how many months will the gym memberships cost the same amount?

$\begin{cases} y = 10x + 15 \\ y = 4x + 40 \end{cases}$ **approximately (4, 55)**

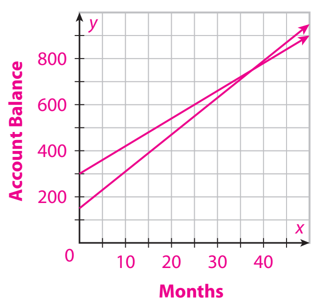
After about 4 months, both gym memberships will cost about $55.



21. Malory is putting money in two savings accounts. Account A started with $150 and Account B started with $300. Malory deposits $16 in Account A and $12 in Account B each month. In how many months will Account A have a balance at least as great as Account B? What will that balance be?

$\begin{cases} y = 16x + 150 \\ y = 12x + 300 \end{cases}$ **approximately (37, 750)**

Account A have a balance at least as great as Account B in about 37 months. The balance will be about $750.



AVOID COMMON ERRORS

Make sure students understand that it is always important to check their solutions. When doing this, they should be sure to substitute the variables into the original equations, not a version of the equation they manipulated while calculating the solution.

PEER-TO-PEER DISCUSSION

Have students discuss with a partner the three types of systems of linear equations (inconsistent; consistent and dependent; and consistent and independent). They should consider the characteristics and number of solutions for each type, then write and graph a system of equations that illustrates each type.

INTEGRATE TECHNOLOGY

Have students use a spreadsheet to model a real-world linear relationship. By changing one or more coefficients, they can easily model variations to a scenario.

JOURNAL

Have students write a journal entry explaining how to solve a system of equations graphically. Students should mention both exact and approximate solutions.

22. Critical Thinking Write *sometimes*, *always*, or *never* to complete the following statements.

a. If the equations in a system of linear equations have the same slope, there are **sometimes** infinitely many solutions for the system.

b. If the equations in a system of linear equations have different slopes, there is **always** one solution for the system.

c. If the equations in a system of linear equations have the same slope and a different y-intercept, there is **never** any solution for the system.

H.O.T. Focus on Higher Order Thinking

23. Critique Reasoning Brad classifies the system below as inconsistent because the equations have the same y-intercept. What is his error?

$$\begin{cases} y = 2x - 4 \\ y = x - 4 \end{cases}$$

Inconsistent systems have the same slope, but different y-intercepts. This system is consistent and independent.

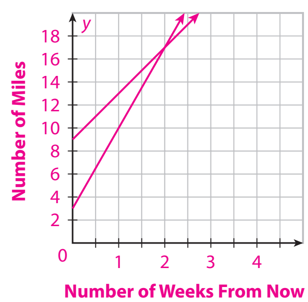
24. Explain the Error Alexa solved the system

$$\begin{cases} 5x + 2y = 6 \\ x - 3y = -4 \end{cases}$$

by graphing and estimated the solution to be about $(1.5, 0.6)$. What is her error? What is the correct answer?

Alexa has reversed the x- and y-values. The correct answer is about $(0.6, 1.5)$.

25. Represent Real-World Problems Cora ran 3 miles last week and will run 7 miles per week from now on. Hana ran 9 miles last week and will run 4 miles per week from now on. The system of linear equations $\begin{cases} y = 7x + 3 \\ y = 4x + 9 \end{cases}$ can be used to represent this situation. Explain what x and y represent in the equations. After how many weeks will Cora and Hana have run the same number of miles? How many miles? Solve by graphing.



The variable x represents the number of weeks from now, and y represents the total number of miles run by each person.

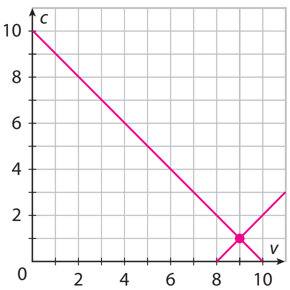
after 2 weeks; 17 miles

Lesson Performance Task

A boat takes 7.5 hours to make a 60-mile trip upstream and 6 hours on the 60-mile return trip. Let v be the speed of the boat in still water and c be the speed of the current. The upstream speed of the boat is $v - c$ and the downstream boat speed is $v + c$.

a. Use the distance formula to write a system of equations relating boat speed and time to distance, one equation for the upstream part of the trip and one for the downstream part.

b. Graph the system to find the speed of the boat in still water and the speed of the current.



c. How long would it take the boat to travel the 60 miles if there were no current?

a. $\begin{cases}(v - c)7.5 = 60\\(v + c)6 = 60\end{cases}$

b. Find the point of intersection of the graphs. The graphs appear to intersect at (9, 1).

Check using both equations.

$(9 - 1)7.5 = 8(7.5) = 60$

$(9 + 1)6 = 10(6) = 60$

The speed v of the boat in still water is 9 mi/h.

The speed of the current is 1 mi/h.

c. $\frac{60 \text{ mi}}{9 \text{ mi/h}} = 6\frac{2}{3}$ **h or 6 h 40 min**

EXTENSION ACTIVITY

Have students consider a boat that is traveling from one side of the river to the opposite bank. Have students research how the current affects the speed of this boat differently from the way it affects a boat traveling upstream or downstream.

Students should find that if the boat heads directly toward the opposite shore, the current will cause it to travel at an angle. They may find that the Pythagorean theorem can be used to determine the resulting speed of the boat, because the current is at a right angle to the direction of the boat.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Communication

MP.3 Have students explain their reasoning for which variable to put on the horizontal axis and which to put on the vertical axis when graphing the system of equations. Students should discover that whether they put v on the horizontal axis and c on the vertical axis or the other way around, the result will be $v = 9$ when $c = 1$.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Reasoning

MP.2 Before students calculate how long it would take for the boat to travel the 60 miles if there were no current, have them predict whether it will be less than 6 hours, between 6 and 7.5 hours, or more than 7.5 hours, and explain their reasoning. Have them compare their calculated results to their predictions.

Scoring Rubric

2 points: Student correctly solves the problem and explains his/her reasoning.

1 point: Student shows good understanding of the problem but does not fully solve or explain his/her reasoning.

0 points: Student does not demonstrate understanding of the problem.