LESSON 11.3

Solving Linear Systems by Adding or Subtracting

Common Core Math Standards

The student is expected to:

COMMON CORE **A-REI.C.6**

Solve systems of linear equations exactly... focusing on pairs of linear equations in two variables.

Mathematical Practices

COMMON CORE **MP.2 Reasoning**

Language Objective

Explain to a partner what eliminating a variable in a system of linear equations means.

ENGAGE

Essential Question: How can you solve a system of linear equations by adding and subtracting?

If the coefficients of the *x*-terms or *y*-terms are the same or are opposites, you can add or subtract the equations to eliminate a variable. Then you can solve for the remaining variable and substitute that value into either of the original equations to solve for the other variable.

PREVIEW: LESSON PERFORMANCE TASK

View the Engage section online. Discuss how much two runners from the same family save by registering as a family instead of as individuals. Then preview the Lesson Performance Task.

Name ______ Class ______ Date ______

11.3 Solving Linear Systems by Adding or Subtracting

Essential Question: How can you solve a system of linear equations by adding and subtracting?

Resource Locker

Explore Exploring the Effects of Adding Equations

Systems of equations can be solved by graphing, substitution, or by a third method, called **elimination.**

(A) Look at the system of linear equations.

$$\begin{cases} 2x - 4y = -10 \\ 3x + 4y = 5 \end{cases}$$

What do you notice about the coefficients of the *y*-terms?

They are opposites or additive inverses.

(B) What is the sum of $-4y$ and $4y$? How do you know?

0; the sum of opposites is 0.

(C) Find the sum of the two equations by combining like terms.

$$\begin{array}{rcrcr} 2x & & -4y & = & -10 \\ +3x & & +4y & = & +5 \\ \hline \mathbf{5x} & + & \mathbf{0} & = & \mathbf{-5} \end{array}$$

(D) Use the equation from Step C to find the value of *x*.

$x =$ **-1**

(E) Use the value of *x* to find the value of *y*. What is the solution of the system?

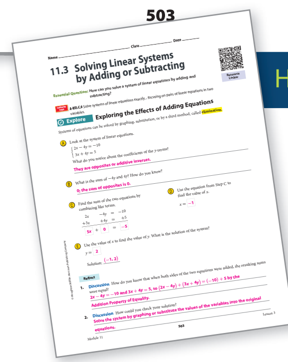
$y =$ **2**

Solution: **$(-1, 2)$**

Reflect

1. **Discussion** How do you know that when both sides of the two equations were added, the resulting sums were equal?
$2x - 4y = -10$ and $3x + 4y = 5$, so $(2x - 4y) + (3x + 4y) = (-10) + 5$ by the Addition Property of Equality.

2. **Discussion** How could you check your solution?
Solve the system by graphing or substitute the values of the variables into the original equations.



HARDCOVER

Turn to Lesson 11.3 in the hardcover edition.

Explain 1 Solving Linear Systems by Adding or Subtracting

The **elimination method** is a method used to solve systems of equations in which one variable is eliminated by adding or subtracting two equations in the system.

Steps in the Elimination Method

1. Add or subtract the equations to eliminate one variable, and then solve for the other variable.
2. Substitute the value into either original equation to find the value of the eliminated variable.
3. Write the solution as an ordered pair.

Example 1 **Solve each system of linear equations using the indicated method. Check your answer by graphing.**

(A) Solve the system of linear equations by adding.

$$\begin{cases} 4x - 2y = 12 \\ x + 2y = 8 \end{cases}$$

Add the equations.

$$\begin{aligned} 4x - 2y &= 12 \\ x + 2y &= 8 \\ \hline 5x + 0 &= 20 \\ 5x &= 20 \\ x &= 4 \end{aligned}$$

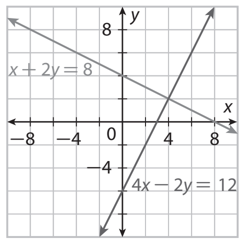
Substitute the value of x into one of the equations and solve for y.

$$\begin{aligned} x + 2y &= 8 \\ 4 + 2y &= 8 \\ 2y &= 4 \\ y &= 2 \end{aligned}$$

Write the solution as an ordered pair.

$(4, 2)$

Check the solution by graphing.



(B) Solve the system of linear equations by subtracting.

$$\begin{cases} 2x + 6y = 6 \\ 2x - y = -8 \end{cases}$$

Subtract the equations.

$$\begin{aligned} 2x + 6y &= 6 \\ -(2x - y &= -8) \\ \hline 0 \quad +7y &= 14 \\ y &= 2 \end{aligned}$$

Substitute the value of y into one of the equations and solve for x.

$$\begin{aligned} 2x - y &= -8 \\ 2x - 2 &= -8 \\ 2x &= -6 \\ x &= -3 \end{aligned}$$

EXPLORE

Exploring the Effects of Adding Equations

INTEGRATE MATHEMATICAL PRACTICES

Focus on Modeling

MP.4 Have students model the linear equations with algebra tiles. They should see that if they add the equations, one of the variables will drop out and they will be able to solve the resulting equation.

QUESTIONING STRATEGIES

How do you know that the result of adding two equations is a true equation? **The left and right sides of an equation are equal to each other. By the Addition Property of Equality, adding one of these equal expressions to each side of a true equation results in a true equation.**

EXPLAIN 1

Solving Linear Systems by Adding or Subtracting

QUESTIONING STRATEGIES

How can you tell if a linear system has a variable that can be eliminated by adding? **The equations will have two like terms that are opposites.**

How can you decide whether to add or subtract to eliminate a variable in a linear system? **If two of the variable terms are opposites, add to eliminate a variable. If two of the variable terms are the same, subtract to eliminate a variable.**

PROFESSIONAL DEVELOPMENT

Integrate Mathematical Practices

This lesson provides an opportunity to address Mathematical Practice **MP.2**, which calls for students to "reason abstractly and quantitatively." Students learn to solve systems of two equations by using elimination. When using this method, students must consider the relationship between the coefficients of the variables to determine whether the equations should be added or subtracted. They connect the results of the algebraic solution method to the graph of the system of equations as they learn to recognize and describe systems that have no solutions or infinitely many solutions.

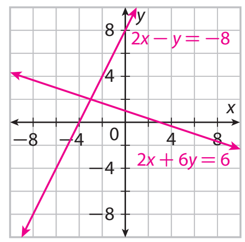
LANGUAGE SUPPORT EL

Connect Vocabulary

Tell students that in the *elimination method*, one variable is eliminated by adding or subtracting. Relate the term *elimination* to what happens in many competitions: teams are eliminated by their opponents. Similarly, a variable is eliminated by its opposite.

Write the solution as an ordered pair.

$(-3, 2)$



Check the solution by graphing.

Reflect

3. Can the system in part A be solved by subtracting one of the original equations from the other? Why or why not?

No; if either of the original equations in the system is subtracted from the other, neither variable will be eliminated.

4. In part B, what would happen if you added the original equations instead of subtracting?

You would get $4x + 5y = -2$, which would not help to solve the system because neither variable would be eliminated.

Your Turn

Solve each system of linear equations by adding or subtracting.

5. $\begin{cases} 2x + 5y = -24 \\ 3x - 5y = 14 \end{cases}$

$$\begin{aligned} 2x + 5y &= -24 \\ 3x - 5y &= 14 \\ \hline 5x + 0 &= -10 \\ 5x &= -10 \\ x &= -2 \\ 2(-2) + 5y &= -24 \\ -4 + 5y &= -24 \\ 5y &= -20 \\ y &= -4 \end{aligned}$$

Solution: $(-2, -4)$

6. $\begin{cases} 3x + 2y = 5 \\ x + 2y = -1 \end{cases}$

$$\begin{aligned} 3x + 2y &= 5 \\ -(x + 2y &= -1) \\ \hline 2x + 0 &= 6 \\ 2x &= 6 \\ x &= 3 \\ x + 2y &= -1 \\ 3 + 2y &= -1 \\ 2y &= -4 \\ y &= -2 \end{aligned}$$

Solution: $(3, -2)$

COLLABORATIVE LEARNING

Peer-to-Peer Activity

Have students work in pairs. Give each pair a system of linear equations to solve. Instruct one student to solve the system of linear equations by elimination. Instruct the other to solve the system by graphing. Have students check that they both got the same solution. Then have students switch roles and repeat the exercise using a different system of linear equations.

Explain 2 Solving Special Linear Systems by Adding or Subtracting

Example 2 Solve each system of linear equations by adding or subtracting.

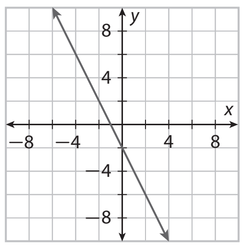
Ⓐ $\begin{cases} -4x - 2y = 4 \\ 4x + 2y = -4 \end{cases}$

Add the equations.

$$\begin{array}{r} -4x - 2y = 4 \\ +4x + 2y = -4 \\ \hline 0 + 0 = 0 \\ 0 = 0 \end{array}$$

The resulting equation is true, so the system has infinitely many solutions.

Graph the equations to provide more information.



The graphs are the same line, so the system has infinitely many solutions.

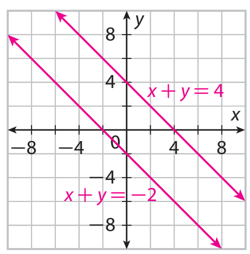
Ⓑ $\begin{cases} x + y = -2 \\ x + y = 4 \end{cases}$

Subtract the equations.

$$\begin{array}{r} x + y = -2 \\ -(x + y = 4) \\ \hline \mathbf{0 + 0 = -6} \\ \mathbf{0 = -6} \end{array}$$

The resulting equation is **false**, so the system has **no** solutions.

Graph the equations to provide more information.



The graph shows that the lines are **parallel** and **do not intersect**.

EXPLAIN 2

Solving Special Linear Systems by Adding or Subtracting

QUESTIONING STRATEGIES

How do you know when a system of linear equations has no solution or infinitely many solutions? **The solution process will result in a false statement for a system with no solutions and a true statement for a system with infinitely many solutions.**

INTEGRATE MATHEMATICAL PRACTICES

Focus on Modeling

MP.4 Remind students that it is possible to model these equations using algebra tiles. Have them predict how the tiles will appear if there are no solutions or infinitely many solutions, then have them try some examples. If there are no solutions, the tiles should be unbalanced after simplifying. If there are infinitely many solutions, there should be zero tiles on both sides after simplifying.

AVOID COMMON ERRORS

Students might think that *infinitely many solutions* means that all ordered pairs are solutions. Remind them that the solutions are the infinite number of ordered pairs that satisfy the equations.

DIFFERENTIATE INSTRUCTION

Graphic Organizers

Have students complete the table to summarize which variable can be eliminated and whether addition or subtraction should be used when solving a system of equations by elimination.

System of Linear Equations Has	Eliminate	By (operation)
x-terms that are opposites	**x**	**adding**
y-terms that are opposites	**y**	**adding**
x-terms that are the same	**x**	**subtracting**
y-terms that are the same	**y**	**subtracting**

EXPLAIN 3

Solving Linear System Models by Adding or Subtracting

QUESTIONING STRATEGIES

After you have found the value of one of the variables, does it matter which equation you substitute the value of that variable into to solve for the other variable? Explain. **No; both variables appear in both equations, and since the solution to the system is the ordered pair that satisfies both equations, the result should be the same no matter which equation you choose.**

INTEGRATE MATHEMATICAL PRACTICES

Focus on Reasoning

MP.2 Remind students to check that the solution makes sense in the context of the problem.

AVOID COMMON ERRORS

Some students may forget to answer the question in a real-world problem after solving the system of equations. Tell students that the solution to the system may not be the final answer, and remind them to make sure they answer the question in the problem.

Your Turn

Solve each system of linear equations by adding or subtracting.

7. $\begin{cases} 4x - y = 3 \\ 4x - y = -2 \end{cases}$

$$\begin{array}{r} 4x - y = 3 \\ -(4x - y = -2) \\ \hline 0 + 0 = 5 \\ 0 = 5 \end{array}$$

The resulting equation is false, so the system has no solutions.

8. $\begin{cases} x - 6y = 7 \\ -x + 6y = -7 \end{cases}$

$$\begin{array}{r} x - 6y = 7 \\ -x + 6y = -7 \\ \hline 0 + 0 = 0 \\ 0 = 0 \end{array}$$

The resulting equation is true, so the system has infinitely many solutions.

Explain 3 Solving Linear System Models by Adding or Subtracting

Example 3 **Solve by adding or subtracting.**

Ⓐ Perfect Patios is building a rectangular deck for a customer. According to the customer's specifications, the perimeter should be 40 meters and the difference between twice the length and twice the width should be 4 meters.

The system of equations $\begin{cases} 2\ell + 2w = 40 \\ 2\ell - 2w = 4 \end{cases}$ can be used to represent this situation, where ℓ is the length and w is the width. What will be the length and width of the deck?

Add the equations.

$$\begin{array}{r} 2\ell + 2w = 40 \\ 2\ell - 2w = 4 \\ \hline 4\ell + 0 = 44 \\ 4\ell = 44 \\ \ell = 11 \end{array}$$

Substitute the value of ℓ into one of the equations and solve for w.

$$\begin{aligned} 2\ell + 2w &= 40 \\ 2(11) + 2w &= 40 \\ 22 + 2w &= 40 \\ 2w &= 18 \\ w &= 9 \end{aligned}$$

Write the solution as an ordered pair.

$(\ell, w) = (11, 9)$

The length of the deck will be 11 meters and the width will be 9 meters.

LANGUAGE SUPPORT EL

Connect Context

Help students eliminate confusion by reminding them of the resources they can use when they are unsure of the meaning of a word. They can consider the context of a problem, use the glossary or a dictionary, think of possible cognates, or ask a friend. Remind them of these synonyms for words in the lesson:
eliminate = remove; opposite = additive inverse; substitute = replace; and false = untrue.

(B) A video game and movie rental kiosk charges $2 for each video game rented, and $1 for each movie rented. One day last week, a total of 114 video games and movies were rented for a total of $177. The system of equations $\begin{cases} x + y = 114 \\ 2x + y = 177 \end{cases}$ represents this situation, where x represents the number of video games rented and y represents the number of movies rented. Find the numbers of video games and movies that were rented.

Subtract the equations.

$$\begin{aligned} x + y &= 114 \\ -(2x + y &= 177) \\ \hline -x + 0 &= -63 \\ -x &= -63 \\ x &= 63 \end{aligned}$$

Substitute the value of x into one of the equations and solve for y.

$$x + y = 114$$
$$63 + y = 114$$
$$y = 51$$

Write the solution as an ordered pair.

(63, 51)

63 video games and **51** movies were rented.

Your Turn

9. The perimeter of a rectangular picture frame is 62 inches. The difference of the length of the frame and twice its width is 1. The system of equations $\begin{cases} 2\ell + 2w = 62 \\ \ell - 2w = 1 \end{cases}$ represents this situation, where ℓ represents the length in inches and w represents the width in inches. What are the length and the width of the frame?

$$\begin{aligned} 2\ell + 2w &= 62 \\ \ell - 2w &= 1 \\ \hline 3\ell + 0 &= 63 \\ 3\ell &= 63 \\ \ell &= 21 \\ 2\ell + 2w &= 62 \end{aligned}$$

$$\begin{aligned} 2(21) + 2w &= 62 \\ 42 + 2w &= 62 \\ 2w &= 20 \\ w &= 10 \end{aligned}$$

The length is 21 inches and the width is 10 inches.

Elaborate

10. How can you decide whether to add or subtract to eliminate a variable in a linear system? Explain your reasoning.

If two of the variable terms are opposites, then you can add to eliminate a variable. If two of the variable terms are the same, then you can subtract to eliminate a variable.

11. **Discussion** When a linear system has no solution, what happens when you try to solve the system by adding or subtracting?

When the equations are added or subtracted to eliminate a variable, the result is a false statement, such as $0 = 9$; this means there is no solution.

12. **Essential Question Check-In** When you solve a system of linear equations by adding or subtracting, what needs to be true about the variable terms in the equations?

The equations must have at least one pair of variable terms that are the same or opposites.

ELABORATE

QUESTIONING STRATEGIES

When the equations in a system have like terms with the same coefficient and you want to eliminate a variable by subtracting, does it matter which equation is subtracted from the other equation? Explain. **No; since the coefficient of the variable is the same in both equations, the variable will be eliminated whether the second equation is subtracted from the first or the first equation is subtracted from the second.**

AVOID COMMON ERRORS

After choosing to use subtraction to solve a linear system of equations, some students may make errors when subtracting negative numbers. Suggest that they use the definition of subtraction to rewrite one of the equations with opposite signs and then add the equations.

SUMMARIZE THE LESSON

How do you solve a system of linear equations by adding or subtracting? **If the equations have like terms whose coefficients are opposites, add the equations to eliminate one variable. If the equations have identical terms, subtract the equations to eliminate one variable. Solve for the value of the remaining variable. Then, substitute that value into either equation and solve for the value of the first variable.**

EVALUATE

Personal Math Trainer

ASSIGNMENT GUIDE

Concepts and Skills	Practice
Explore Exploring the Effects of Adding Equations	Exercises 1, 22
Example 1 Solving Linear Systems by Adding or Subtracting	Exercises 2–9, 23–24
Example 2 Solving Special Linear Systems by Adding or Subtracting	Exercises 10–15
Example 3 Solving Linear System Models by Adding or Subtracting	Exercises 16–21, 25

Evaluate: Homework and Practice

Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

1. Which method of elimination would be best to solve the system of linear equations? Explain.

$$\begin{cases} \frac{1}{2}x + \frac{3}{4}y = -10 \\ -x - \frac{3}{4}y = 1 \end{cases}$$

The addition method would be best because the *y*-values are opposites, or additive inverses.

Solve each system of linear equations by adding or subtracting.

2. $\begin{cases} 3x + 2y = 10 \\ 3x - y = 22 \end{cases}$

$$\begin{aligned} 3x + 2y &= 10 \\ -(3x - y &= 22) \\ \hline 0 + 3y &= -12 \\ y &= -4 \\ 3x - (-4) &= 22 \\ x &= 6 \end{aligned}$$

Solution: (6, −4)

3. $\begin{cases} -2x + y = 3 \\ 3x - y = -2 \end{cases}$

$$\begin{aligned} -2x + y &= 3 \\ 3x - y &= -2 \\ \hline x + 0 &= 1 \\ x &= 1 \\ 3(1) - y &= -2 \\ y &= 5 \end{aligned}$$

Solution: (1, 5)

4. $\begin{cases} x + y = 5 \\ x - 3y = 3 \end{cases}$

$$\begin{aligned} x + y &= 5 \\ -(x - 3y &= 3) \\ \hline 0 + 4y &= 2 \\ y &= 0.5 \\ x + 0.5 &= 5 \\ x &= 4.5 \end{aligned}$$

Solution: (4.5, 0.5)

5. $\begin{cases} 7x + y = -4 \\ 2x - y = 1 \end{cases}$

$$\begin{aligned} 7x + y &= -4 \\ 2x - y &= 1 \\ \hline 9x + 0 &= -3 \\ x &= -\frac{1}{3} \\ 7\left(-\frac{1}{3}\right) + y &= -4 \\ y &= -\frac{5}{3} \end{aligned}$$

Solution: $\left(-\frac{1}{3}, -\frac{5}{3}\right)$

6. $\begin{cases} -5x + y = -3 \\ 5x - 3y = -1 \end{cases}$

$$\begin{aligned} -5x + y &= -3 \\ 5x - 3y &= -1 \\ \hline 0 - 2y &= -4 \\ y &= 2 \\ 5x - 3(2) &= -1 \\ x &= 1 \end{aligned}$$

Solution: (1, 2)

7. $\begin{cases} 2x + y = -6 \\ -5x + y = 8 \end{cases}$

$$\begin{aligned} 2x + y &= -6 \\ -(-5x + y &= 8) \\ \hline 7x + 0 &= -14 \\ x &= -2 \\ 2(-2) + y &= -6 \\ y &= -2 \end{aligned}$$

Solution: (−2, −2)

8. $\begin{cases} 6x - 3y = 15 \\ 4x - 3y = -5 \end{cases}$

$$\begin{aligned} 6x - 3y &= 15 \\ -(4x - 3y &= -5) \\ \hline 2x + 0 &= 20 \\ x &= 10 \\ 6(10) - 3y &= 15 \\ y &= 15 \end{aligned}$$

Solution: (10, 15)

9. $\begin{cases} 8x - 6y = 36 \\ -2x + 6y = 0 \end{cases}$

$$\begin{aligned} 8x - 6y &= 36 \\ -2x + 6y &= 0 \\ \hline 6x + 0 &= 36 \\ x &= 6 \\ -2(6) + 6y &= 0 \\ y &= 2 \end{aligned}$$

Solution: (6, 2)

Exercise	Depth of Knowledge (D.O.K.)	COMMON CORE Mathematical Practices
1	**1** Recall of Information	**MP.2** Reasoning
2–18	**2** Skills/Concepts	**MP.2** Reasoning
19	**2** Skills/Concepts	**MP.4** Modeling
20	**2** Skills/Concepts	**MP.2** Reasoning
21	**2** Skills/Concepts	**MP.4** Modeling
22	**2** Skills/Concepts	**MP.2** Reasoning

10. $\begin{cases} \frac{1}{2}x - \frac{7}{9}y = -\frac{20}{3} \\ -\frac{1}{2}x + \frac{7}{9}y = 6\frac{2}{3} \end{cases}$

$$\begin{array}{r} \frac{1}{2}x - \frac{7}{9}y = -\frac{20}{3} \\ -\frac{1}{2}x + \frac{7}{9}y = 6\frac{2}{3} \\ \hline 0 + 0 = 0 \\ 0 = 0 \end{array}$$

Infinitely many solutions

11. $\begin{cases} -10x + 2y = -7 \\ -10x + 2y = -2 \end{cases}$

$$\begin{array}{r} -10x + 2y = -7 \\ -(-10x + 2y = -2) \\ \hline 0 + 0 = -5 \\ 0 = -5 \end{array}$$

No solution

12. $\begin{cases} -2x + 5y = 7 \\ 2x - 5y = -7 \end{cases}$

$$\begin{array}{r} -2x + 5y = 7 \\ +2x - 5y = -7 \\ \hline 0 + 0 = 0 \\ 0 = 0 \end{array}$$

Infinitely many solutions

13. $\begin{cases} x + y = 0 \\ -x - y = 0 \end{cases}$

$$\begin{array}{r} x + y = 0 \\ -x - y = 0 \\ \hline 0 + 0 = 0 \\ 0 = 0 \end{array}$$

Infinitely many solutions

14. $\begin{cases} -5x - y = -3 \\ -5x - y = -2 \end{cases}$

$$\begin{array}{r} -5x - y = -3 \\ -(-5x - y = -2) \\ \hline 0 + 0 = -1 \\ 0 = -1 \end{array}$$

No solution

15. $\begin{cases} ax - by = c \\ ax - by = c \end{cases}$

$$\begin{array}{r} ax - by = c \\ -(ax - by = c) \\ \hline 0 + 0 = 0 \\ 0 = 0 \end{array}$$

Infinitely many solutions

16. The sum of two numbers is 65, and the difference of the numbers is 27. The system of linear equations $\begin{cases} x + y = 65 \\ x - y = 27 \end{cases}$ represents this situation, where x is the larger number and y is the smaller number. Solve the system to find the two numbers.

$$\begin{array}{r} x + y = 65 \\ x - y = 27 \\ \hline 2x + 0 = 92 \\ x = 46 \\ x + y = 65 \\ 46 + y = 65 \\ y = 19 \end{array}$$

The larger number is 46 and the smaller number is 19.

VISUAL CUES

Suggest that students circle the coefficient of the variable that is to be eliminated in each equation, including the plus or minus sign. This visual cue can help them remember to subtract the equations if the coefficients are the same and to add the equations if the coefficients are opposites.

Exercise	Depth of Knowledge (D.O.K.)	COMMON CORE Mathematical Practices
23	3 Strategic Thinking H.O.T.	MP.4 Modeling
24	3 Strategic Thinking H.O.T.	MP.3 Logic
25	3 Strategic Thinking H.O.T.	MP.6 Precision

AVOID COMMON ERRORS

Point out to students that when they use the elimination method, they must line up both the like terms and the equal signs vertically.

KINESTHETIC EXPERIENCE

To check solutions by graphing, create a coordinate grid on the floor using painter's tape. Have students help you decide where to place a tape line for one of the equations. Then have another student walk the line for the second equation. The student should stop when encountering the tape line for the first equation. The class should help the student identify that position's coordinates and check the solution. If the lines are parallel, the student should note that the paths will never cross.

17. A rectangular garden has a perimeter of 120 feet. The length of the garden is 24 feet greater than twice the width. The system of linear equations $\begin{cases} 2\ell + 2w = 120 \\ \ell - 2w = 24 \end{cases}$ represents this situation, where ℓ is the length of the garden and w is its width. Find the length and width of the garden.

$$\begin{aligned} 2\ell + 2w &= 120 \\ \underline{\ell - 2w} &= \underline{\ 24} \\ 3\ell + 0 &= 144 \\ \ell &= 48 \\ 2\ell + 2w &= 120 \\ 2(48) + 2w &= 120 \\ 96 + 2w &= 120 \\ 2w &= 24 \\ w &= 12 \end{aligned}$$

The length of the garden is 48 feet and the width is 12 feet.

18. The sum of two angles is 90°. The difference of twice the larger angle and the smaller angle is 105°. The system of linear equations $\begin{cases} x + y = 90 \\ 2x - y = 105 \end{cases}$ represents this situation where x is the larger angle and y is the smaller angle. Find the measures of the two angles.

$$\begin{aligned} x + y &= 90 \\ \underline{2x - y} &= \underline{105} \\ 3x + 0 &= 195 \\ 3x &= 195 \\ x &= 65 \\ x + y &= 90 \\ 65 + y &= 90 \\ y &= 25 \end{aligned}$$

The larger angle is 65° and the smaller angle is 25°.

19. Max and Sasha exercise a total of 20 hours each week. Max exercises 15 hours less than 4 times the number of hours Sasha exercises. The system of equations $\begin{cases} x + y = 20 \\ x - 4y = -15 \end{cases}$ represents this situation, where x represents the number of hours Max exercises and y represents the number of hours Sasha exercises. How many hours do Max and Sasha exercise per week?

$$\begin{aligned} x + y &= 20 \\ \underline{-(x - 4y} &\underline{= -15)} \\ 0 + 5y &= 35 \\ y &= 7 \\ x + y &= 20 \\ x + 7 &= 20 \\ x &= 13 \end{aligned}$$

Max exercises 13 hours a week and Sasha exercises 7 hours a week.

20. The sum of the digits in a two-digit number is 12. The digit in the tens place is 2 more than the digit in the ones place. The system of linear equations $\begin{cases} x + y = 12 \\ x - y = 2 \end{cases}$ represents this situation, where x is the digit in the tens place and y is the digit in the ones place. Solve the system to find the two-digit number.

$$\begin{aligned} x + y &= 12 \\ \underline{x - y} &= \underline{\ 2} \\ 2x + 0 &= 14 \\ x &= 7 \\ x + y &= 12 \\ 7 + y &= 12 \\ y &= 5 \end{aligned}$$

The number is 75.

21. A pool company is installing a rectangular pool for a new house. The perimeter of the pool must be 94 feet, and the length must be 2 feet more than twice the width.

The system of linear equations $\begin{cases} 2\ell + 2w = 94 \\ \ell = 2w + 2 \end{cases}$ represents this situation, where ℓ is the length and w is the width. What are the dimensions of the pool?

$$\begin{aligned} 2\ell + 2w &= 94 \\ \ell - 2w &= 2 \\ \hline 3\ell + 0 &= 96 \\ 3\ell &= 96 \\ \ell &= 32 \\ 2\ell + 2w &= 94 \\ 2(32) + 2w &= 94 \\ 64 + 2w &= 94 \\ 2w &= 30 \\ w &= 15 \end{aligned}$$

The pool is 32 feet long and 15 feet wide.

22. Use one solution, no solutions, or infinitely many solutions to complete each statement.

a. When the solution of a system of linear equations yields the equation $4 = 4$, the system has **infinitely many solutions**.

b. When the solution of a system of linear equations yields the equation $x = 4$, the system has **one solution**.

c. When the solution of a system of linear equations yields the equation $0 = 4$, the system has **no solutions**.

H.O.T. Focus on Higher Order Thinking

23. Multiple Representations You can use subtraction to solve the system of linear equations shown.

$$\begin{cases} 2x + 4y = -4 \\ 2x - 2y = -10 \end{cases}$$

Instead of subtracting $2x - 2y = -10$ from $2x + 4y = -4$, what equation can you add to get the same result? Explain.

You can add $-2x + 2y = 10$ since subtracting is the same as adding the opposite.

COMMUNICATING MATH

Have students discuss when to use each of the methods they have learned for solving systems of equations. Graphing can be used for linear systems that have integer solutions. Substitution can be used when one of the equations has a variable with a coefficient of 1. Adding or subtracting can be used when the coefficient of a variable in one equation is the same as, or the opposite of, the coefficient of the same variable in the other equation.

JOURNAL

Have students write a journal entry explaining how to solve a system of linear equations by eliminating a variable. They should include how you know whether to add or subtract the equations.

24. Explain the Error Liang's solution of a system of linear equations is shown. Explain Liang's error and give the correct solution.

$$\begin{cases} 3x - 2y = 12 \\ -x - 2y = -20 \end{cases}$$

$$\begin{array}{r} 3x - 2y = 12 \\ \underline{-x - 2y = -20} \\ 2x = -8 \\ x = -4 \\ 3x - 2y = 12 \\ 3(-4) - 2y = 12 \\ -12 - 2y = 12 \\ -2y = 24 \\ y = -12 \end{array}$$

Solution: $(-4, -12)$

Liang added the two equations but subtracted the *y*-terms. The equations should be subtracted. The solution is $(8, 6)$.

$$\begin{array}{r} 3x - 2y = 12 \\ \underline{-(-x - 2y = -20)} \\ 4x = 32 \\ x = 8 \\ 3(8) - 2y = 12 \\ 24 - 2y = 12 \\ -2y = -12 \\ y = 6 \end{array}$$

25. Represent Real-World Problems For a school play, Rico bought 3 adult tickets and 5 child tickets for a total of \$40. Sasha bought 1 adult ticket and 5 child tickets for a total of \$25.

The system of linear equations $\begin{cases} 3x + 5y = 40 \\ x + 5y = 25 \end{cases}$ represents this situation, where x is the cost of an adult ticket and y is the cost of a child ticket. How much will Julia pay for 5 adult tickets and 3 child tickets?

$$\begin{array}{r} 3x + 5y = 40 \\ \underline{-(x + 5y = 25)} \\ 2x + 0 = 15 \\ x = 7.5 \\ x + 5y = 25 \\ 7.5 + 5y = 25 \\ 5y = 17.5 \\ y = 3.5 \end{array}$$

$5(7.5) + 3(3.5) = 48$

Julia will pay \$48.

Lesson Performance Task

A local charity run has a Youth Race for runners under the age of 12. The entry fee is \$5 for an individual or \$4 each for two runners from the same family. Carter is collecting the registration forms and fees. After everyone has registered, he picks up the cash box and finds a dollar on the ground. He checks the cash box and finds that it contains \$200 and the registration slips for 47 runners. Does the dollar belong in the cash box or not? Explain your reasoning. (Hint: You can use the system of equations $i + f = 47$ and $5i + 4f = 200$, where i equals the number of individual tickets and f equals the number of family tickets.)

Solving the system gives the solution $i = 12$ and $f = 35$. However, this result is not possible because f must be an even number since family registrations were sold only as pairs. If we add the \$1 to the total cash, we can rewrite the second equation as $5i + 4f = 201$. Solving the new system yields the solution $i = 13$ and $f = 34$, so the dollar does belong in the cash box. (Note that some students may simply reason from the initial solution that they can swap one family registration for one individual registration and get a total of \$201.)

EXTENSION ACTIVITY

On an index card, have students write a riddle that can be represented by a system of equations whose solution can be found by elimination. For example: *The sum of one number and another number is 5. The difference between the two numbers is 1.* On the back of the card, have students write the solution and show the system of equations used to find it. Then have students trade cards and solve each other's riddles.

QUESTIONING STRATEGIES

? Family tickets were sold only when two runners from the same family registered at the same time. What does this tell you about the total number of family tickets sold? **It must be an even number.**

AVOID COMMON ERRORS

Some students may try to add the two given equations to solve the system by elimination. Remind them that the elimination method only works when the equations have like terms whose coefficients are the same or opposites. They should recognize that they can solve this system by substitution.

KINESTHETIC EXPERIENCE

Have students use play money, a cash box, and ticket stubs to act out the situation so they can better visualize how different numbers of individual and family tickets sold would result in different amounts of money in the cash box.

Scoring Rubric

2 points: Student correctly solves the problem and explains his/her reasoning.

1 point: Student shows good understanding of the problem but does not fully solve or explain his/her reasoning.

0 points: Student does not demonstrate understanding of the problem.